STA408
Design and Analysis of Experiments
Lesson 3

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Randomized Complete Block Design

Testing the effects of two independent variables or factors on one *dependent variable.*

Hypothesis Testing – General Procedure

Procedure Table

Solving Hypothesis-Testing Problems (Traditional Method)

Step 1	State the hypotheses and identify the claim.
Step 2	Find the critical value(s) from the appropriate table
Step 3	Compute the test value.
Step 4	Make the decision to reject or not reject the null hypothesis.
Step 5	Summarize the results.

Main Points

- Experimental material is divided into groups or blocks
  - Why
- Each block contains a complete set of treatments
- Treatments are assigned randomly
  - Randomizations is restricted within block

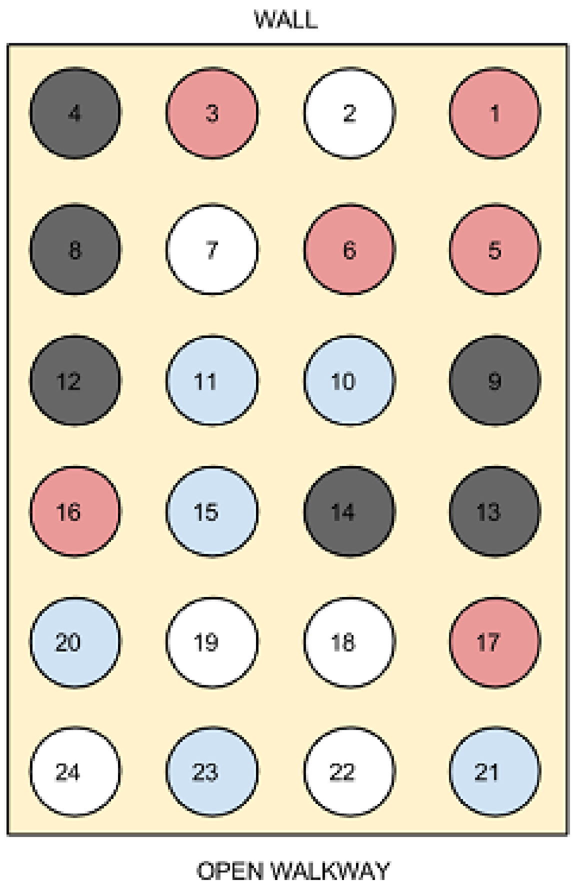
Main Points

- A new randomization is done for each block/group

- It has different layouts: 2X3, 3X5, etc

- The RCBD is perhaps the most frequently used experimental design

CRD vs RCBD



Fertilizer 1 - Blue

Fertilizer 2 - Red

Fertilizer 3 - Black

No Fertilizer - White (control)

Fertilizer 1 - Blue

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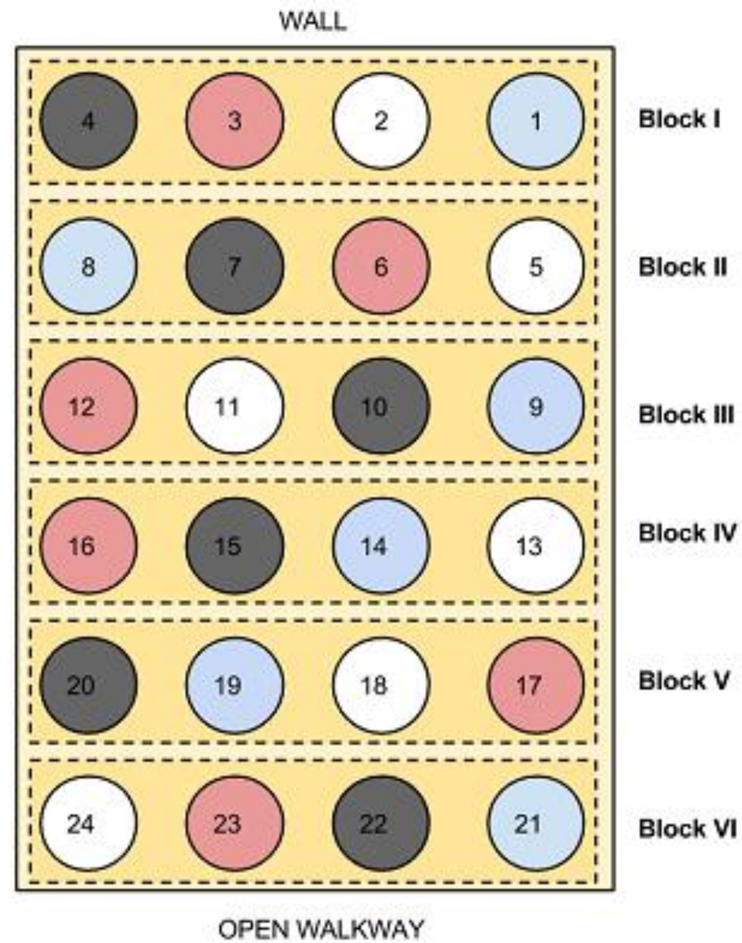

May look like

The Agricultural Research Field

The Agricultural Research Field

Experimental Layout

Allot treatment randomly

BLOCK - I				

BLOCK - II				

BLOCK - III				

Statistical Model

- The liner model for this design is:

$$Y_{ij} = \mu + B_i + T_j + e_{ij}$$

- Where:

μ = True mean effect

T = Effect of treatment

B = Effect of block/groups

e = effect of random error

Data Computations Table

Block (Groups)	Treatments						Block	
	1	*2*	…	*j*	…	*k*	Total	Mean
1	Y_{11}	Y_{12}					$B_{1\bullet}$	$\bar{Y}_{1\bullet}$
2								
i								
r								
Total	$B_{\bullet 1}$							
Means	$\bar{Y}_{\bullet 1}$							

Partitioning of Variations

$$\begin{bmatrix}\text{Total}\\ \text{Variation}\end{bmatrix}=\begin{bmatrix}\text{Variation due to}\\ \text{Treatment}\end{bmatrix}+\begin{bmatrix}\text{Variation due to}\\ \text{Block/Grouping}\end{bmatrix}+\begin{bmatrix}\text{Variation due to}\\ \text{Unknown factors}\end{bmatrix}$$

Total SS = Treatment SS + Block SS + Error SS

$$\sum_i\sum_j(Y_{ij}-\bar{Y})^2 = r\sum_j(\bar{Y}_{\cdot j}-\bar{Y}_{\cdot\cdot})^2 + k\sum_i(\bar{Y}_{i\cdot}-\bar{Y}_{\cdot\cdot})^2 + \sum_i\sum_j(Y_{ij}-\bar{Y}_{\cdot j}-\bar{Y}_{i\cdot}-\bar{Y}_{\cdot\cdot})$$

AVOVA Table for RCB

Source of Variation s.o.v	Degree of Freedom d.f	SS = Sum of Squares	MS = Mean Squares	F -test
(Variations due to) **Treatment**	K-1	$SSTr = r\sum_{j}(\bar{Y}_{\cdot j} - \bar{Y}_{\cdot\cdot})^2$	$S_t^2 = \frac{SSTr}{k-1}$	$F_t = \frac{S_t^2}{S_e^2} = \frac{MST}{MSE}$
(Variations due to) **Blocks**	r- 1	$SSB = k\sum_{i}(\bar{Y}_{i\cdot} - \bar{Y}_{\cdot\cdot})^2$	$S_b^2 = \frac{SSB}{r-1}$	$F_b = \frac{S_b^2}{S_e^2} = \frac{MSB}{MSE}$
(Variations due to) **Error**	(k-1) (r-1)	SSE = By Subtraction	$S_e^2 = \frac{SSTr}{(r-1)(k-1)}$	
Total	rk-1	$SST = \sum_{i}\sum_{j}(Y_{ij} - \bar{Y}_{\cdot\cdot})^2$	---	

Example

- Four verities of wheat ere tested in randomized complete clock design in four replications/blocks. Yield in kg/plot is shown in the table below.
- Test the hypothesis that there is no difference in the means of four varieties.

Blocks	Varieties of wheat			
	V1	V2	V3	V4
I	2	5	4	1
II	2	3	3	1
III	4	6	6	2
IV	1	4	2	3

Testing Hypothesis – Example

Step 1

Stat the hypothesis.

$H_0 : Tj = 0$ (There is NO difference in the means of four varieties of wheat)
$H_1 : Tj \neq 0$ (There is difference in the means of four varieties)
(Not ALL the four means are equal)

$H_0 : B_i = 0$ (There is NO difference in the means of four varieties of wheat)
$H_1 : B_i \neq 0$ (There is difference in the means of four varieties)
(Not ALL the four means are equal)

Step 2

We set the **level of significance (alpha)** $\boldsymbol{\alpha = 0.05}$ **(5%)**

Testing Hypothesis – Example

Step 3

The test-statistic (formula) to be used is:

$$F = \frac{S_t^2}{S_e^2} = \frac{MS(Treatment)}{MS(Error)}$$

It follows F-distribution with degree of freedom

df = v_1 = from treatment = k-1

df = v_2 = from error = (k-1)(r-1)

Computations

Blocks	Varieties (Treatment)						
	V1	V2	V3	V4	*Block Total*	$\left(\frac{Block}{Total}\right)^2$	Sum of Square of each value
I	2	5	4	1	2+5+4+1 = 12	12*12 = 144	$2^2+5^2+4^2+1^2 = 46$
II	2	3	3	1	9	81	23
III	4	6	6	2	18	324	92
IV	1	4	2	3	10	100	30
$\left(\frac{\text{Varities}}{\text{Total}}\right)$							
$\left(\frac{\text{Varities}}{\text{Total}}\right)^2$							
Sum of Square of each value							

Computations

$$\text{SST} = \text{ Sum of Squares of Total } = \Sigma\Sigma Y^2 - \frac{T_{\square\square}}{n}$$

$$= \left(2^2 + 5^2 + 4^2 + \ldots + 3^2\right) - \frac{\left(2+5+4+\ldots+3\right)^2}{16}$$

$$= 191 - \frac{(49)^2}{16} = 40.94$$

$$\text{SSTr} = \text{ Sum of Squares of Treatment(varities)} = -\Sigma\Sigma Y^2 - \frac{T_{\square\square}}{n}$$

$$= \left(2^2 + 5^2 + 4^2 + \ldots + 3^2\right) - \frac{\left(2+5+4+\ldots+3\right)^2}{16}$$

$$= 191 - \frac{(49)^2}{16} = 40.94$$

Computations

$$\text{SSTr} = \text{ Sum of Squares of Treatment(varieties)} = \frac{T^2}{r} - \frac{T_{\square\square}}{n}$$

$$= \frac{679}{4} - \frac{(49)^2}{16} = 169.75 - 150.06 = 19.69$$

$$\text{SSB} = \text{ Sum of Squares of Block} = \frac{B^2}{r} - \frac{T_{\square\square}}{n}$$

$$= \frac{649}{4} - \frac{(49)^2}{16} = 162.25 - 150.06 = 12.19$$

$$\text{SSE} = \text{ Sum of Squares of Error} = SST - SSTr - SSB$$

$$= 40.96 - 19.69 - 12.19 = 9.06$$

ANOVA Table

Source of Variation s.o.v	Degree of Freedom d.f	SS = Sum of Squares	MS = Mean Squares	Computed F -test
(Variations due to) **Treatment**	4-1 = 3	19.69	19.69 / 3 = 6.56	$F_t = \frac{6.56}{1.01} = 6.50$
(Variations due to) **Blocks**	4 – 1 = 3	12.19	12.19 / 3 = 4.06	$F_b = \frac{4.06}{1.01} = 4.02$
(Variations due to) **Error**		9.06	9.06 / 9 = 1.01	
Total	(4*4) – 1 =15	40.96	---	

F – Table Value

Table H *(continued)*

d.f.D.: degrees of freedom, denominator	α = 0.05												
	d.f.N.: degrees of freedom, numerator												
	1	2	3	4	5	6	7	8	9	10	12	15	20
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.

Comparison and Conclusion

Step 5

$$Calculated\ \ Value \text{ of F (treatment)} = 6.56$$

$$\text{Critical/Table Value of F} = 3.86$$

$$\Rightarrow 6.56 > 3.86$$

$$\Rightarrow F_t > F(Table / CriticalValue)$$

Conclusion and Summarizing:

Since the computed value of **F (treatment) is GREATER THAN the F Table value, So it** falls in the critical region. We reject our null hypothesis and may conclude that the means of four varieties of wheat are significantly different.

Comparison and Conclusion

Step 5

$$\textit{Calculated Value} \text{ of F (Block/Group)} = 4.06$$

$$\text{Critical/Table Value of F} = 3.86$$

$$\Rightarrow 4.06 > 3.86$$

$$\Rightarrow F_t > F(Table / Critical Value)$$

Conclusion and Summarizing:

Since the computed value of **F (Block/group) is GREATER THAN the F Table value, So it** falls in the critical region. We reject our null hypothesis and may conclude that the **BLOCKING or GROUPING was effective.**

Another example

- The following is the plan of a filed layout testing four varieties A, B, C and D of wheat in each of % blocks. The plot yields in Kgs are also indicated.

Block I	
D	29.3
B	33.3
C	30.8
A	32.3

Block II	
B	33.0
A	34.0
C	34.3
D	26.0

Block III	
D	29.8
A	34.3
B	36.3
C	35.3

Block IV	
B	36.8
A	35.00
D	28.0
C	32.3

Block V	
D	28.8
C	35.8
B	34.5
A	36.5

Computations

Blocks	Varieties (Treatment)						
	V1	V2	V3	V4	*Block Total*	$\left(\begin{matrix} Block \\ Total \end{matrix}\right)^2$	Sum of Square of each value
I							
II							
III							
IV							
$\left(\begin{matrix} \text{Varities} \\ \text{Total} \end{matrix}\right)$							
$\left(\begin{matrix} \text{Varities} \\ \text{Total} \end{matrix}\right)^2$							
Sum of Square of each value							

ANOVA Table

Source of Variation s.o.v	Degree of Freedom d.f	SS = Sum of Squares	MS = Mean Squares	Computed F -test
(Variations due to) **Treatment**				
(Variations due to) **Blocks**				
(Variations due to) **Error**				
Total				

Advantages

- The source of **extraneous** variation is controlled by grouping the experimental material and hence the estimate of the experimental error is decreased.
- The design is flexible. Any number of replications may be run and any number of treatments may be tested.
- The experiment can be set up easily
- The statistical analysis is simple and straightforward.
- It is easy to adjust the missing observations

Disadvantages

- It control the variability only in one direction
- It is not suitable design when the number of treatments is very large or when the blocks are not homogeneous.

- Extraneous Variables are **undesirable** variables that influence the relationship between the variables that an experimenter is examining.
- Another way to think of this, is that these are variables that influence the outcome of an experiment, though they are not the variables that are actually of interest. These variables are undesirable because they add error to an experiment. A major goal in research design is to decrease or control the influence of extraneous variables as much as possible.

Summary

- Experimental material is divided into groups or blocks
- Each block contains a complete set of treatments
- Treatments are assigned randomly for each block
- Data is organized in ANOVA table
- F-test is used for testing
- Rest of the testing procedure is same.

The END