STA 408: DESIGN AND ANALYSIS OF EXPERIEMENTS

S. M. Muslim Raza
Instructor Statistics
Virtual university of Pakistan

Contents

1. Basic concepts of ANOVA

2. CR Design

Analysis of Variance: A Conceptual Overview

- Analysis of Variance (ANOVA) can be used to test for the equality of three or more population means.
- Data obtained from observational or experimental studies can be used for the analysis.
- We want to use the sample results to test the following hypotheses:

$$H_0\!: \mu_1 = \mu_2 = \mu_3 = \cdot\ \cdot\ \cdot = \mu_k$$

H_a: Not all population means are equal

Analysis of Variance: A Conceptual Overview

H_0: $\mu_1 = \mu_2 = \mu_3 = \cdot \ \cdot \ \cdot = \mu_k$

H_a: Not all population means are equal

- If H_0 is rejected, we cannot conclude that *all* population means are different.
- Rejecting H_0 means that at least two population means have different values.

Analysis of Variance: A Conceptual Overview

- Assumptions for Analysis of Variance

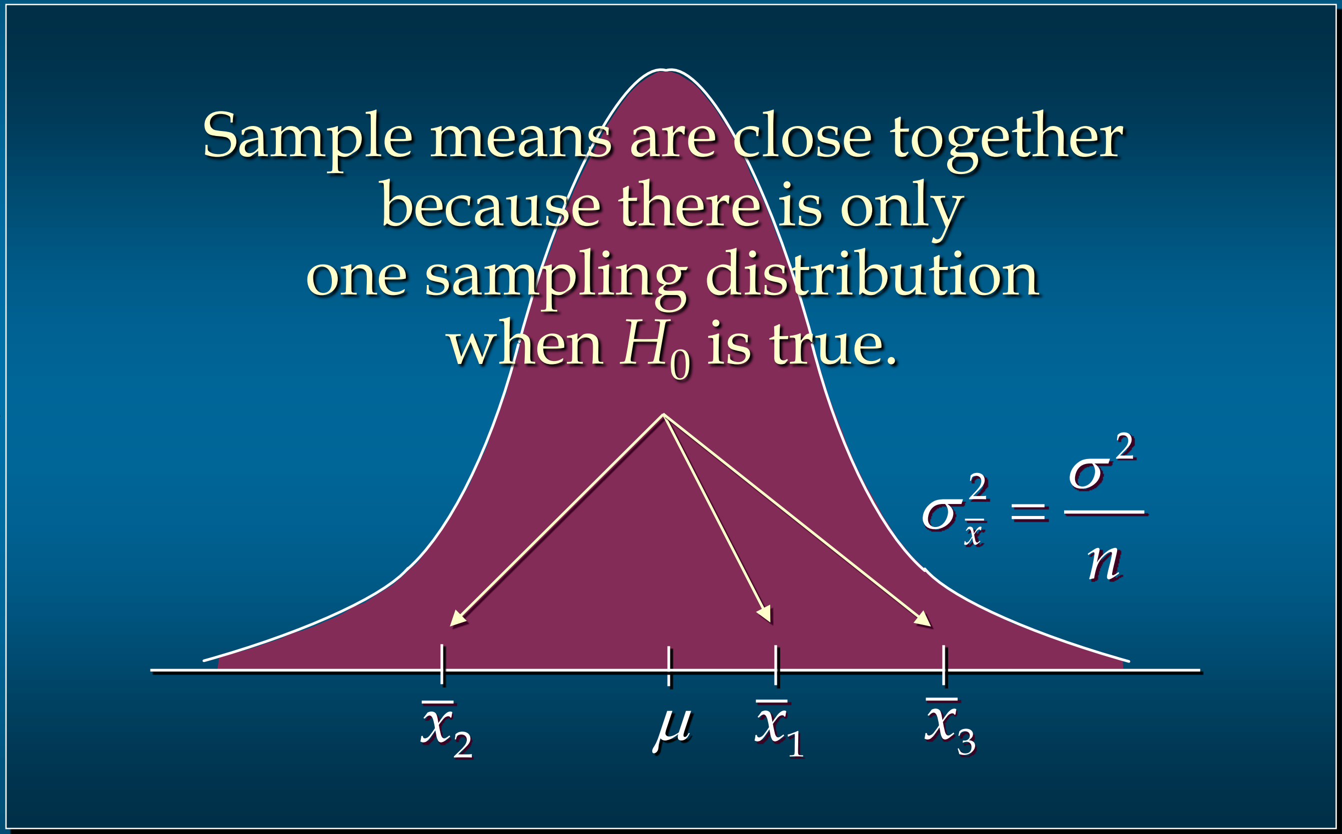
  - For each population, the response (dependent) variable is normally distributed.

  - The variance of the response variable, denoted σ^2, is the same for all of the populations.

  - The observations must be independent.

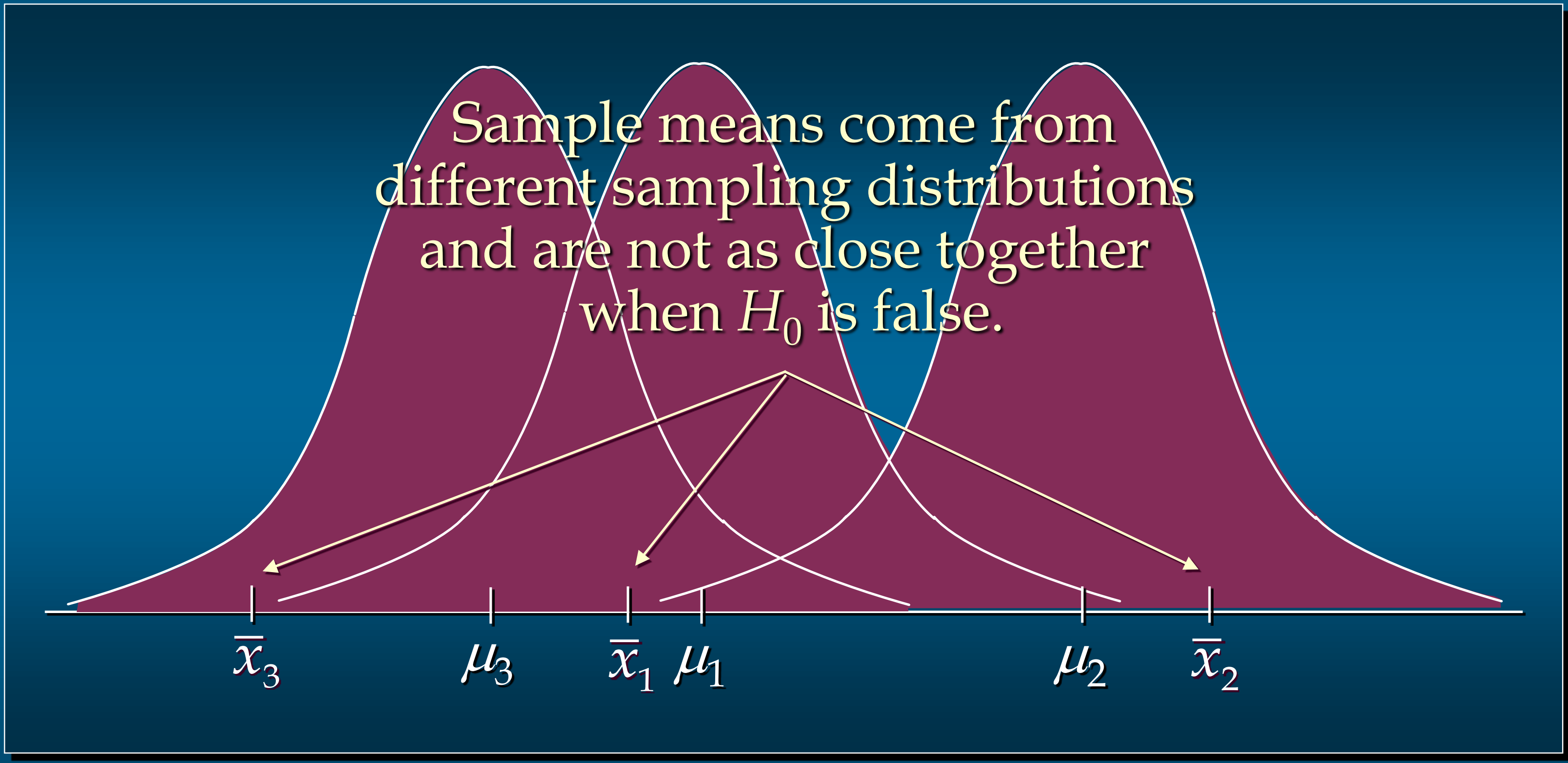
Analysis of Variance: A Conceptual Overview

- Sampling Distribution of $\bar{x}$ Given H_0 is True



Analysis of Variance: A Conceptual Overview

- Sampling Distribution of $\bar{x}$ Given H_0 is False



Analysis of Variance

- Between-Treatments Estimate of Population Variance
- Within-Treatments Estimate of Population Variance
- Comparing the Variance Estimates: The *F* Test
- ANOVA Table

Between-Treatments Estimate of Population Variance σ^2

- The estimate of σ^2 based on the variation of the sample means is called the mean square due to treatments and is denoted by MSTR.

$$\text{MSTR} = \frac{\sum_{j=1}^{k} n_j(\bar{x}_j - \bar{\bar{x}})^2}{k-1}$$

Denominator is the degrees of freedom associated with SSTR

Numerator is called the sum of squares due to treatments (SSTR)

Within-Treatments Estimate of Population Variance σ^2

- The estimate of σ^2 based on the variation of the sample observations within each sample is called the <u>mean square error</u> and is denoted by <u>MSE</u>.

$$\text{MSE} = \frac{\sum_{j=1}^{k} (n_j - 1) s_j^2}{n_T - k}$$

Numerator is called the <u>sum of squares due to error</u> (SSE)

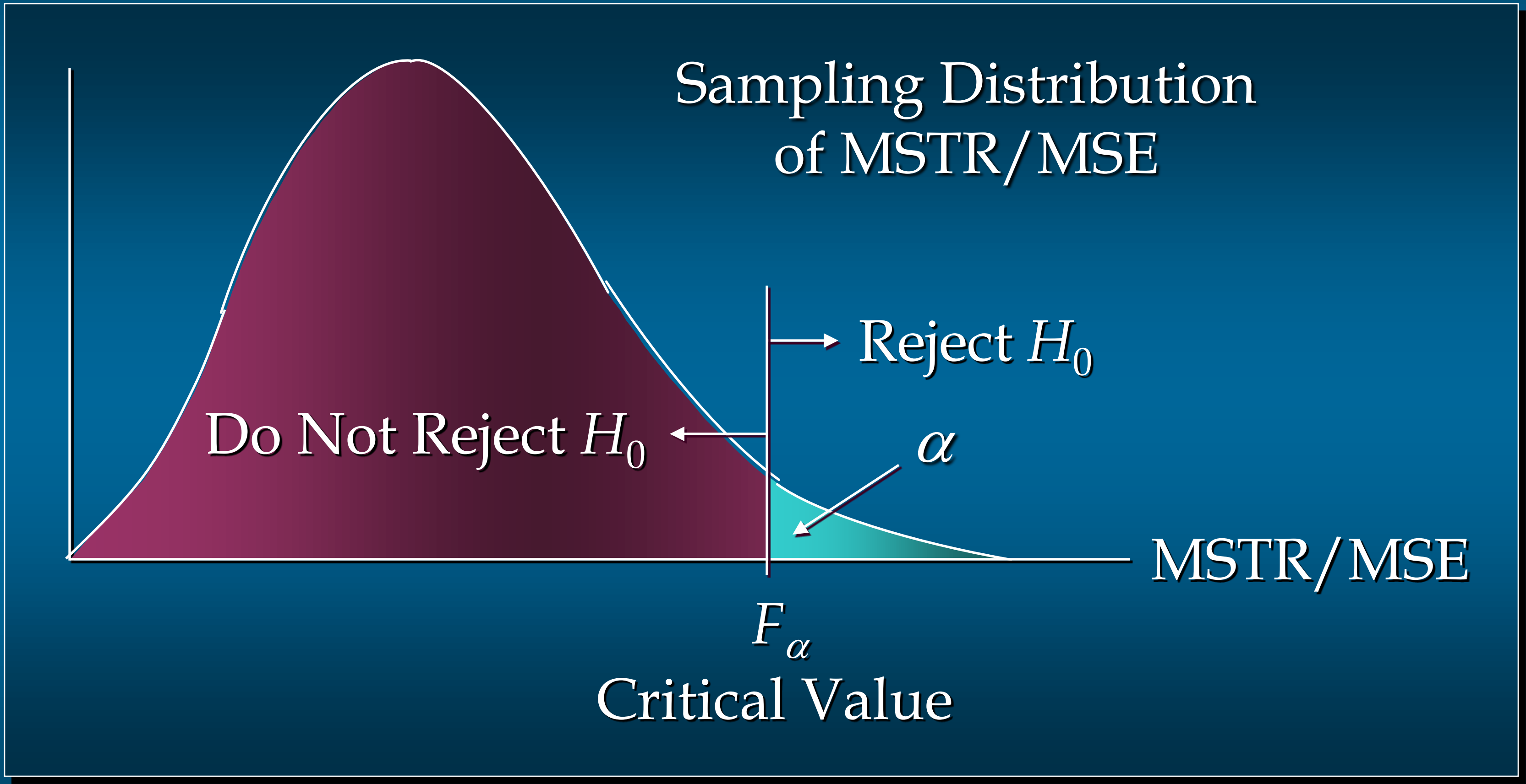
Denominator is the <u>degrees of freedom</u> associated with SSE

Comparing the Variance Estimates: The *F* Test

- If the null hypothesis is true and the ANOVA assumptions are valid, the sampling distribution of MSTR/MSE is an *F* distribution with MSTR d.f. equal to k - 1 and MSE d.f. equal to n_T - k.
- If the means of the k populations are not equal, the value of MSTR/MSE will be inflated because MSTR overestimates σ^2.
- Hence, we will reject H_0 if the resulting value of MSTR/MSE appears to be too large to have been selected at random from the appropriate *F* distribution.

Comparing the Variance Estimates: The F Test

- Sampling Distribution of MSTR/MSE



Test for the Equality of k Population Means

- Hypotheses

$$H_0\text{: } \mu_1 = \mu_2 = \mu_3 = \cdot\ \cdot\ \cdot = \mu_k$$

H_a: Not all population means are equal

- Test Statistic

$$F = \text{MSTR}/\text{MSE}$$

Test for the Equality of k Population Means

- Rejection Rule
  - p-value Approach: Reject H_0 if p-value $\leq \alpha$
  - Critical Value Approach: Reject H_0 if $F \geq F_\alpha$

where the value of F_α is based on an F distribution with k - 1 numerator d.f. and $n_T - k$ denominator d.f.

ANOVA Table

- The ANOVA table decomposes the variance into two components (sources): a between-group component (treatments) and a within-group component (error). The F-ratio, which in this case equals 15.6234, is a ratio of the between-group estimate to the within-group estimate. Since the P-value of the F-test is less than 0.05, there is a statistically significant difference between the means from one level to another at the 95.0% confidence level.

ANOVA Table for hours by brand

Source	*Sum of Squares*	*Df*	*Mean Square*	*F-Ratio*	*P-Value*
Between groups	1.08917	3	0.363057	15.62	0.0000
Within groups	0.557714	24	0.0232381		
Total (Corr.)	1.64689	27			

Completely randomized design (CRD)

- A factor is a variable that the experimenter has selected for investigation (the independent variable).
- A treatment is a level of a factor.
- Experimental units are the objects of interest in the experiment.
- A completely randomized design is an experimental design in which the treatments are randomly assigned to the experimental units.

COMPLETELY RANDOMIZED DESIGN

A completely randomized (CR) design, which is the simplest type of the basic designs, may be defined as a design in which the treatments are assigned to experimental units completely at random, that is the randomization is done without any restrictions. The design is completely flexible, i.e. any number of treatments and any number of units per treatment may be used. Moreover, the number of units per treatment need not be equal. A completely randomized design is considered to be most useful in situations where (i) the experimental units are homogeneous, (ii) the experiments are small such as laboratory experiments and (iii) some experimental units are likely to be destroyed or to fail to respond.

CRD

- When the samples of experimental units for each treatment are random and independent of each other
- Design is used to compare the treatment means:

$$H_0 : \mu_1 = \mu_2 = \ldots \mu_k$$

$$H_a : At\ least\ two\ of\ the\ treatment\ means\ differ$$

- The hypotheses are tested by comparing the differences between the treatment means.
- Test statistic is calculated using measures of variability within treatment groups and measures of variability between treatment groups

Steps for Conducting an Analysis of Variance (ANOVA) for a Completely Randomized Design:

- 1- Assure randomness of design, and independence, randomness of samples
- 2- Check normality, equal variance assumptions
- 3- Create ANOVA summary table
- 4- Conduct multiple comparisons for pairs of means as necessary/desired

Assumptions

1- Normality:

You can check on normality using

1- Normal probability plot

2- Kolmogorve test

2- Constant variance:

You can check on homogeneity of variances using

1- Histogram Plot

2- leven's test.

CRD

ANOVA Summary Table for a Completely Randomized Design

Source	*df*	*SS*	*MS*	*F*
Treatments	$k-1$	SST	$MST=\dfrac{SST}{k-1}$	$\dfrac{MST}{MSE}$
Error	$n-k$	SSE	$MSE=\dfrac{SSE}{n-k}$	
Total	$n-1$	$SS(Total)$		

Advantages of a CRD

- Very flexible design (i.e. number of treatments and replicates is only limited by the available number of experimental units).
- Statistical analysis is simple compared to other designs.
- Loss of information due to missing data is small compared to other designs due to the larger number of degrees of freedom for the error source of variation.

Disadvantages

- If experimental units are not homogeneous and you fail to minimize this variation using blocking, there may be a loss of precision.
- Usually the least efficient design unless experimental units are homogeneous.
- Not suited for a large number of treatments.

Experimental Layout:

An example of the experimental layout for a completely randomized design (CR) using four treatments A, B, C and D, each repeated 3 times, is given below:

Example:

An experiment was conducted to compare the yields of three varieties of potatoes. Each variety was assigned at random to equal-size plots, four times. The yields were as follows:

Variety		
A	B	C
23	18	16
26	28	25
20	17	12
17	21	14

Test the hypothesis that the three varieties of potatoes are not different in the yielding capabilities.

Example:

Three fertilizer treatments A, B, C each applied to seven plots of strawberry plants, resulted in the following weights of crops (Ib/plot):

A	24	18	18	29	22	17	15
B	46	39	37	50	44	45	30
C	32	30	26	41	36	28	27

Perform the analysis of variance to test the hypothesis of no difference in the treatment effects. Use a 0.01 level of significance.

Solution:

Formulation of Hypothesis:

$H_0: \mu_1 = \mu_2 = \mu_3$

$H_1:$ *At least two means are not equal.*

Level of Significance:

$\alpha = 0.05.$

Test Statistics:

$$F = \frac{MS\ Treatment}{MS\ Error}$$

Calculation:

A	24	18	18	29	22	17	15	**143**
B	46	39	37	50	44	45	30	**291**
C	32	30	26	41	36	28	27	**220**
Total								**654**

Here $T_1 = 143, T_2 = 291, T_3 = 220$

$T_{..} = 654$

Total SS= $\sum\sum X_{ij}^2 - CF$

CF = $T^2_{..}/N$, where N= nk= 7(3) = 21.

CF= $(654)^2/21$ =20367.43

Total SS= $\sum\sum X_{ij}^2 - CF$

Total SS= $24^2 + 18^2 + 18^2 + \ldots + 28^2 - 20367.43$

$$= 22520 - 20367.43 = 2152.57$$

Treat SS= $\dfrac{\sum_{j=1}^{k} T_{.j}^2}{n} - CF$

$$\text{Treat SS} = \frac{\sum_{j=1} T_{.j}^2}{n} - CF$$

$$= \frac{1}{7}\left[(143)^2 + (291)^2 + (220)^2\right] - 20367.43$$

$$= 21932.86 - 20367.43 = 1564.43$$

Error SS= Total SS - Treat SS

$$= 2152.57 - 1564.43 = 587.14$$

<table>
<tr><th>Source of Variation</th><th>Sum of squares</th><th>Degree of freedom</th><th>Mean squares</th><th>F ratio</th></tr>
<tr><td>Treatment</td><td>1565.43</td><td>k-1 = 2</td><td>=1564.43/2
=782.72</td><td rowspan="2">F=782.72/32.62
=24.00</td></tr>
<tr><td>Error</td><td>587.14</td><td>N − k = 18</td><td>=587.14/18
=32.62</td></tr>
<tr><td>Total</td><td>2152.57</td><td>N − 1 = 20</td><td></td><td></td></tr>
</table>

Critical Region:

Reject Ho, if

$$F_{cal} \geq F_{\alpha(k-1, N-k)}$$

$$24.00 \geq F_{0.05(2,18)}$$

24.00 > 3.55

We reject Ho.

Conclusion:

Since the value 24.00 > 3.55 fall in the critical region, so we reject Ho or also we conclude that the there is a difference between the treatment effects.

THANK YOU