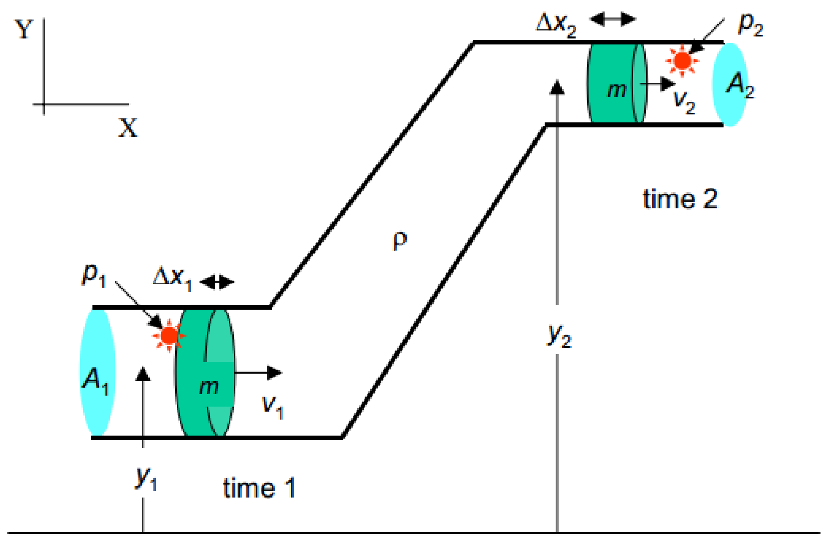
Dear students, here is the complete derivation of Bernoulli's equation.

Derivation of Bernoulli's equation



As we know that

Density = mass /volume

Density is represented by ρ (rho)

As, Volume

V = length* width* height

Also Area = length*height,

Thus V = A*width = A*x_1

Hence

mass = density*volume = $\rho A_1 \Delta x_1$

$m = \rho A_1 \Delta x_1$

Mass element m moves from (1) to (2)

$m = \rho A_1 \Delta x_1 = \rho A_2 \Delta x_2 = \rho \Delta V$ where $\Delta V = A_1 \Delta x_1 = A_2 \Delta x_2$

Equation of continuity $A V$ = constant

$A_1 v_1 = A_2 v_2 \quad A_1 > A_2 \Rightarrow v_1 < v_2$

Since $v_1 < v_2$ the mass element has been accelerated by the net force

$$F_1 - F_2 = p_1 A_1 - p_2 A_2$$

Mass element has an increase in KE

$$\Delta K = ½\, m\, v_2^2 - ½\, m\, v_1^2 = ½\, \rho\, \Delta V\, v_2^2 - ½\, \rho\, \Delta V\, v_1^2$$

Mass element has an increase in GPE

$$\Delta U = m\, g\, y_2 - m\, g\, y_1 = \rho\, \Delta V\, g\, y_2 = \rho\, \Delta V\, g\, y_1$$

The increase in KE and GPE comes from the net work done on the mass element by the forces F_1 and F_2 (the sample of mass m, in moving from a region of higher pressure to a region of lower pressure, has positive work done on it by he surrounding fluid)

$$W_{net} = F_1 \Delta x_1 - F_2 \Delta x_2 = p_1 A_1 \Delta x_1 - p_2 A_2 \Delta x_2$$
$$W_{net} = p_1 \Delta V - p_2 \Delta V = \Delta K + \Delta U$$

$$p_1 \Delta V - p_2 \Delta V =$$
$$\tfrac{1}{2} \rho \Delta V v_2^2 - \tfrac{1}{2} \rho \Delta V v_1^2 + \rho \Delta V g y_2 - \rho \Delta V g y_1$$

Rearranging

$$p_1 + \tfrac{1}{2} \rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2} \rho v_2^2 + \rho g y_2$$

! This is Bernoulli's equation.

We can also write, for any point along a flow tube or streamline

$$p + \tfrac{1}{2} \rho v^2 + \rho g y = \text{constant}$$

This is Bernoulli's theorem.