MTH721 (Spring 2018) Assignment No. 1

Maximum Marks: 25

Due Date: May 28, 2018

INSTRUCTIONS

Please read the following instructions before attempting the solution of this assignment:

- **To solve this assignment, you should have good command over 1 to 13 Lectures.**
- **Try to get the concepts, consolidate your concepts which you learn in these lectures with these questions.**
- **Upload assignments properly through LMS. No Assignment will be accepted through email.**
- **Write your ID on the top of your solution file.**
- **Do not use colorful backgrounds in your solution files.**
- **Use Math Type or Equation Editor etc. for mathematical symbols and equations.**
- **Zero marks will be awarded for a copied solution. That is if the solution files of any two students are found same, both of them will be awarded zero marks. Therefore, try to make solution by yourself and protect your work from other students.**
- **Avoid copying the solution from book (or internet); you must solve the assignment yourself.**
- **Also remember that you are supposed to submit your assignment in Word format any other format like scanned images, HTML etc. will not be accepted**

Note: Attempt all the following questions.

Question: 1	**Marks: 5**

Let *I, J* and *K* are ideals in ring *R*. Show that $I:(\mathrm{J}+\mathrm{K})=(\mathrm{I}:\mathrm{J})\cap(\mathrm{I}:\mathrm{K})$.

Question: 2	**Marks: 5**

Show that the binary operation * defined on the set $\mathbb{R}^{+}$ of nonzero real numbers defined by letting $a*b=a/b$. is not a group. Explain your answer.

Question: 3	**Marks: 5**

Let $I=(x^3,x^2y,y^3)$ and $J=(x,y)$ be an ideals in a polynomial ring $R=K[\mathrm{x},\mathrm{y}]$. Calculate $I:J$.

Question: 4	**Marks: 5**

Show that the map $f:\mathbb{Z}\to\mathbb{Z}_7$ defined by $f(\mathrm{a})=a \bmod 7$ for all $a\in\mathbb{Z}$ is a ring homomorphism. Find also the kernel of f.

Question: 5	**Marks: 5**

Find all ideals I of $\mathbb{Z}_{12}$. In each case compute $\mathbb{Z}_{12}/I$; that is, find a known ring to which the quotient ring is isomorphic.