

Q. List all topologies on $X = \{1,2,3\}$ which consists of exactly four members.

Solution.

$$\tau_1 = \{\emptyset, \{1\}, \{2,3\}, X\}, \tau_2 = \{\emptyset, \{2\}, \{1,3\}, X\}, \tau_3 = \{\emptyset, \{3\}, \{1,2\}, X\}$$

$$\tau_4 = \{\emptyset, \{1\}, \{1,2\}, X\}, \tau_5 = \{\emptyset, \{2\}, \{1,2\}, X\}, \tau_6 = \{\emptyset, \{1\}, \{1,3\}, X\},$$

$$\tau_7 = \{\emptyset, \{3\}, \{1,3\}, X\}, \tau_8 = \{\emptyset, \{2\}, \{2,3\}, X\}, \tau_9 = \{\emptyset, \{3\}, \{2,3\}, X\}$$