solve:

$$I = \int_0^{2\pi}\int_0^1 [2(2\cos\theta)^2 - (2\sin\theta)^3]2dzd\theta$$

Solution: (here is the solution of your problem of module 54)

First integrate with respect to z

$$I = 2\int_0^{2\pi} [2(2\cos\theta)^2 z - (2\sin\theta)^3 z]|_0^3 d\theta$$

$$I = 2\int_0^{2\pi} [6(2\cos\theta)^2 - 3(2\sin\theta)^3] d\theta$$

$$I = 48\int_0^{2\pi} [(\cos\theta)^2 - (\sin\theta)^3] d\theta$$

consider $\sin^3\theta = \sin\theta\sin^2\theta = \sin\theta(1-\cos^2\theta)$

Also

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$ and $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

Replacing these values in the integral we have

$$\begin{aligned}
I &= 48\int_0^{2\pi} [(\frac{1+\cos 2\theta}{2}) - (\sin\theta(1-\cos^2\theta))]d\theta, \\
&= 48\int_0^{2\pi} [(\frac{1+\cos 2\theta}{2}) - (\sin\theta(1-\cos^2\theta))]d\theta, \\
&= 48\int_0^{2\pi} [(\frac{1+\cos 2\theta}{2}) - (\sin\theta - \cos^2\theta\sin\theta)]d\theta, \\
&= 48\int_0^{2\pi} [\frac{1}{2} + \frac{\cos 2\theta}{2} - \sin\theta + \cos^2\theta\sin\theta]d\theta, \\
&= 48[\frac{1}{2}\theta + \frac{\sin 2\theta}{4} + \cos\theta - \frac{\cos^3\theta}{3}]|_0^{2\pi}, \\
&= 48\pi
\end{aligned}$$