$$\left.\begin{aligned} 6x_1 - 3x_2 + x_3 &= 11 \\ 2x_1 + x_2 - 8x_3 &= -15 \\ x_1 - 7x_2 + x_3 &= 10 \end{aligned}\right\} \rightarrow (1)$$

$\Rightarrow$ **Step 1:-**

$$\left.\begin{aligned} 11 - 6x_1 + 3x_2 - x_3 &= 0 \rightarrow R_1 \\ -15 - 2x_1 - x_2 + 8x_3 &= 0 \rightarrow R_3 \\ 10 - x_1 + 7x_2 - x_3 &= 0 \rightarrow R_2 \end{aligned}\right\} \rightarrow (2)$$

Note:-

If (1) is written in matrix form, then

$$\begin{bmatrix} \boxed{6} & -3 & 1 \\ 2 & \boxed{1} & -8 \\ 1 & -7 & \boxed{1} \end{bmatrix} \rightarrow \text{Diagonal.}$$

Above is not true arrangement because we want largest coefficient on the diagonal. So interchange the last two rows. We have.

$$\begin{array}{c} \begin{bmatrix} \boxed{6} & -3 & 1 \\ 1 & \boxed{-7} & 1 \\ 2 & 1 & \boxed{-8} \end{bmatrix} \end{array} \begin{array}{l} \rightarrow R_1, R_2 \\ \rightarrow \text{Diagonal} \\ \rightarrow R_3. \end{array}$$

Step II:- Start with an initial guesses

$X_1 = X_2 = X_3 = 0$; then ① $\Rightarrow$.

$R_1 = 1$

$R_2 = 10$

$R_3 = -15$

Largest Residual is "$R_3 = -15$"

Step III:-

$\Rightarrow dx_3 = -R_3/a_{33}$

$dx_3 = -(-15)/8$.

$$\boxed{dx_3 = 1.875}$$

Step IV:-

New guess is

$X_1 = 0$; $X_2 = 0$; $X_3 = 1.875$

Second iteration:-

Putting $x_1 = 0$, $x_2 = 0$ and

$x_3 = 1.875$, the ① $\Rightarrow$.

$R_1 = 11 - 1.875 = 9.125$

$R_2 = 10 - 1.875 = 8.125$

$R_3 = -15 + 8(1.875) = 0$

$\Rightarrow$ Largest Residual $\boxed{R_1 = 9.125}$

$$\Rightarrow dx_1 = -\frac{R_1}{a_{11}}$$

$$= +\left(\frac{9.125}{6}\right).$$

$$\boxed{dx_1 = 1.5208.}$$

Third iteration

New guesses is

$$X_1 = 1.5208$$

$$X_2 = 0$$

$$X_3 = 1.875$$

Repeat the same procedure like iteration 2 and at the 8th iteration, you got

$$R_1 = 0.0178$$

$$R_2 = 0.0659$$

$$R_3 = 0.0003$$

and

$$X_1 = 1.0017$$

$$X_2 = -0.9901$$

$$X_3 = 2.0017.$$