CROUT'S method:-

$$\left.\begin{aligned} x_1 + 2x_2 + 5x_3 &= -11 \\ x_2 &= 1 \\ 2x_1 + 9x_2 + 11x_3 &= -20 \end{aligned}\right\} \to (1) \Rightarrow AX = b$$

method:- Let

$$A = LU$$

$$\begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 0 \\ 2 & 9 & 11 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 0 \\ 2 & 9 & 11 \end{bmatrix} = \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12} + l_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31} & l_{31}u_{12} + l_{32} & l_{31}u_{13} + l_{32}u_{23} + l_{33} \end{bmatrix}$$

$$\Rightarrow \boxed{l_{11} = 1}, \boxed{l_{21} = 0}, \boxed{l_{31} = 2}$$

$$l_{11}u_{12} = 2 \Rightarrow \boxed{u_{12} = 2}, \quad l_{11}u_{13} = 5 \Rightarrow \boxed{u_{13} = 5}$$

$$l_{21}u_{12} + l_{22} = 1 \Rightarrow \boxed{l_{22} = 1}$$

$$l_{21}u_{13} + l_{22}u_{23} = 0 \Rightarrow \boxed{u_{23} = 0}$$

$$l_{31}u_{12} + l_{32} = 9 \Rightarrow 2 \times 2 + l_{32} = 9 \Rightarrow \boxed{l_{32} = 5}$$

$$l_{31}u_{13} + l_{32}u_{23} + l_{33} = 11$$

$$\Rightarrow 2 \times 5 + 5 \times 0 + l_{33} = 11 \Rightarrow \boxed{l_{33} = 1}$$

So,

$$L=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 5 & 1 \end{bmatrix}, \quad U=\begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

then by system (1); we have:

$$AX = b$$

$$\Rightarrow LUX = b$$

$$\Rightarrow L(UX) = b$$

Let:

$$\Rightarrow UX = Y$$

$$\Rightarrow LY = b$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 5 & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -11 \\ 1 \\ -20 \end{bmatrix}.$$

$$\Rightarrow \begin{bmatrix} y_1 \\ y_2 \\ 2y_1+5y_2+y_3 \end{bmatrix} = \begin{bmatrix} -11 \\ 1 \\ -20 \end{bmatrix}.$$

$$\Rightarrow \boxed{y_1 = -11; \quad y_2 = 1}$$

$$\Rightarrow 2(-11) + 5(1) + y_3 = -20.$$

$$\Rightarrow \boxed{y_3 = -3}.$$

$$\Rightarrow \quad Y = \begin{bmatrix} -11 \\ 1 \\ -3 \end{bmatrix}$$

Now

$$UX = Y.$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -11 \\ 1 \\ -3 \end{bmatrix}$$

$$\Rightarrow \quad x_1 + 2x_2 + 5x_3 = -11$$

$$x_2 = 1$$

$$x_3 = -3.$$

$$\Rightarrow \quad x_1 = -11 - 2(1) - 5(-3).$$

$$= -11 - 2 + 15.$$

$$= 2.$$

$$\Rightarrow \boxed{X = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}}.$$

Checking.

Put $x_1 = 2$; $x_2 = 1$ and $x_3 = -3$ in (1), we have:

$$2 + 2(1) + 5(-3) = -11$$

$$\Rightarrow 2 + 2 - 15 = -11$$

$$\boxed{-11 = -11}$$