# Closed form solution

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$$\frac{dy}{dt} - y(t) = t \tag{1}$$

We will find its solution using integrating factor technique; the integrating factor may be determined as:

$$\begin{aligned} I.F. &= e^{\int(-1)dt} \\ &= e^{-t}. \end{aligned} \tag{2}$$

Now multiplying the Eq. (1) with $e^{-t}$, we have

$$\begin{aligned} e^{-t}\frac{dy}{dt} - e^{-t}y(t) &= e^{-t}t \\ \frac{d}{dt}(e^{-t}y) &= e^{-t}t, \end{aligned} \tag{3}$$

integrating Eq. (3), we have

$$\int \left(\frac{d}{dt}(e^{-t}y)\right) dt = \int (e^{-t}t)dt, \tag{4}$$

integrating the right hand side with integration by parts, we have

$$\begin{aligned} e^{-t}y &= -te^{-t} - e^{-t} + C \\ y(t) &= -t - 1 + Ce^{t}, \end{aligned} \tag{5}$$

Which is the required form.