Practice Questions for Lecture No. 1-3

Question 1:

Convert the decimal number 80 into its binary equivalent.

Solution:

2	80	Remainder
2	40	0
2	20	0
2	10	0
2	5	0
2	2	1
	1	0

Thus, $(80)_{10} = (1010000)_2$.

Question 2:

Convert the binary number $(11001100)_2$ to its decimal equivalent.

Solution:

$$(11001100)_2 = 1\times 2^7 + 1\times 2^6 + 0\times 2^5 + 0\times 2^4 + 1\times 2^3 + 1\times 2^2 + 0\times 2^1 + 0\times 2^0$$
$$= 128 + 64 + 0 + 0 + 8 + 4 + 0 + 0$$
$$= 204$$

Thus,

$(11001100)_2 = (204)_{10}$

Question 3:

Find the relative error when $\sqrt{17}$ is considered upto four decimal places.

Solution:

True value = 4.123105625

Computed value =4.1231

$$|Error| = |0.0000056| = 0.0000056$$

$$\text{Relative Error} = \frac{|\text{Error}|}{|\text{True Value}|} = \frac{0.0000056}{4.123105625}$$
$$= 0.00000136$$

Question 4:

Find the interval in which atleast one root of the equation $x^3 - x^2 - 2x + 1 = 0$ lies.

Solution:

Given equation is

$$x^3 - x^2 - 2x + 1 = 0$$

Using intermediate value property, we will find any two values of x such that the function has open signs on these two values. It will give us the confirmation that atleast one root lies between these two values.

We have

$$f(0) = (0)^3 - (0)^2 - 2(0) + 1 = 1$$
$$f(1) = (1)^3 - (1)^2 - 2(1) + 1 = -1$$

Since $f(0).\text{f}(1) < 0$ this gives us the confirmation that atleast one root lies between x=0 and x= 1, thus, the desired interval is [0,1].

Question 5:

Find the real root of the equation $x^4 - x - 10 = 0$ in the interval $[1, 2]$ by bisection method upto two iterations.

Solution:

Given that

$$f(x) = x^4 - x - 10 \quad [1, 2]$$
$$f(1) = (1)^4 - 1 - 10 = -10$$
$$f(2) = (2)^4 - 2 - 10 = 4$$

Here, $f(1)f(2) < 0$ so the root lies between x = 1 and x =2.

Let's take $x_0 = 1, \ x_1 = 2$

$$x_2 = \frac{1+2}{2} = 1.5$$
$$f(1.5) = (1.5)^4 - (1.5) - 10 = -6.4375$$
$$Now,$$
$$f(2).f(1.5) < 0$$
$$so,$$
$$x_3 = \frac{1.5+2}{2} = 1.75$$
$$f(1.75) = (1.75)^4 - (1.75) - 10 = -2.37$$

Thus, the approximate root of the given equation is 1.75.
