Here it is given the differential equation: $\frac{dy}{dx} = t + y$, with initial condition of $y(1) = 0$, which can also written as $y_0 = 0$ at $t = 1$.

Now to apply the Taylor' series, we need to find the some successive derivatives.

$\because \frac{dy}{dx} = t + y$ or $y' = t + y$, then the next derivative w.r.t t is $y'' = 1 + y'$ and others are $y''' = y''$ and $y'''' = y''$.

So all these values at $t = 1$ are:

$y_0 = y(t_0) = y(1) = 0$, which is equivalent to given $y(1) = 0$.

For y_0',put $t_0 = 1$ and $y_0 = 0$ in $y' = t + y$

$\implies y_0' = 1$

For y_0'', put $t_0 = 1$ and $y_0' = 1$ in $y'' = 1 + y'$

$\implies y_0'' = 1 + 1 = 2$

For y_0''', put $y_0'' = 2$ in $y''' = y''$

$\implies y_0''' = 2$. Similarly we get $y_0'''' = 2$.