

Since the Truncation error for  $g(x) = \cos x$  is given by:

$$\text{Truncation error} \leq \frac{x^{2n+2}}{(2n+2)!}$$

Now remember that *truncation error is always evaluated at some specific value in the given domain* (which unfortunately is not mentioned in lecture-02 at page-08), so here choose  $x = \pi$ , then  $\frac{x^{2n+2}}{(2n+2)!}$  becomes say:  $f(n) = \frac{(\pi)^{2n+2}}{(2n+2)!}$ .

Now truncation error *upto m-decimal places* is given by;

$$f(n) < \frac{1}{2} \times 10^{-m}$$

In particular for *5-decimal places*:

$$f(n) < \frac{1}{2} \times 10^{-5}$$

$$\implies \frac{(\pi)^{2n+2}}{(2n+2)!} < \frac{1}{2} \times 10^{-5}$$

Now we verify the values of  $n$  for which the above inequality is satisfied. By using MAPLE, we can see that it is not true for  $n = 1, 2, \dots, 6$ , but for  $n = 7$  it starts working.

$$\left\{ \begin{array}{l} \text{istru}e(f(1) < 0.5 \times 10^{-5}) = \text{false} \\ \text{istru}e(f(2) < 0.5 \times 10^{-5}) = \text{false} \\ \text{istru}e(f(3) < 0.5 \times 10^{-5}) = \text{false} \\ \text{istru}e(f(4) < 0.5 \times 10^{-5}) = \text{false} \\ \text{istru}e(f(5) < 0.5 \times 10^{-5}) = \text{false} \\ \text{istru}e(f(6) < 0.5 \times 10^{-5}) = \text{false} \\ \\ \text{istru}e(f(7) < 0.5 \times 10^{-5}) = \text{true} \end{array} \right.$$

$\therefore$  seven terms are needed in the expansion  $g(x) = \cos x$  at  $x = \pi$  for the *desired accuracy upto 5 decimal places*.