

If $A = \begin{pmatrix} \cos^2 \alpha & \cos \alpha \sin \alpha \\ \cos \alpha \sin \alpha & \sin^2 \alpha \end{pmatrix}$ and $B = \begin{pmatrix} \cos^2 \beta & \cos \beta \sin \beta \\ \cos \beta \sin \beta & \sin^2 \beta \end{pmatrix}$
that $\alpha = 5\frac{\pi}{2} + \beta$, then show that $AB = BA = 0$.

Since we have given $\alpha = \frac{5\pi}{2} + \beta$
 $\implies \cos a = \cos\left(\frac{5\pi}{2} + \beta\right) = \cos\frac{5\pi}{2} \cos \beta - \sin\frac{5\pi}{2} \sin \beta = (0) \cos \beta - (1) \sin \beta = -\sin \beta$
 $\implies \sin a = \sin\left(\frac{5\pi}{2} + \beta\right) = \sin\frac{5\pi}{2} \cos \beta + \cos\frac{5\pi}{2} \sin \beta = (1) \cos \beta + (0) \sin \beta = \cos \beta$

Now

$$\begin{aligned} AB &= \begin{pmatrix} \cos^2 \alpha & \cos \alpha \sin \alpha \\ \cos \alpha \sin \alpha & \sin^2 \alpha \end{pmatrix} \begin{pmatrix} \cos^2 \beta & \cos \beta \sin \beta \\ \cos \beta \sin \beta & \sin^2 \beta \end{pmatrix} \\ &= \begin{pmatrix} (-\sin \beta)^2 & -\cos \beta \sin \beta \\ -\cos \beta \sin \beta & \cos^2 \beta \end{pmatrix} \begin{pmatrix} \cos^2 \beta & \cos \beta \sin \beta \\ \cos \beta \sin \beta & \sin^2 \beta \end{pmatrix} \\ &= \begin{pmatrix} (-\sin \beta)^2 & -\cos \beta \sin \beta \\ -\cos \beta \sin \beta & \cos^2 \beta \end{pmatrix} \begin{pmatrix} \cos^2 \beta & \cos \beta \sin \beta \\ \cos \beta \sin \beta & \sin^2 \beta \end{pmatrix} \\ &= \begin{pmatrix} \sin^2 \beta & -\cos \beta \sin \beta \\ -\cos \beta \sin \beta & \cos^2 \beta \end{pmatrix} \begin{pmatrix} \cos^2 \beta & \cos \beta \sin \beta \\ \cos \beta \sin \beta & \sin^2 \beta \end{pmatrix} \\ &= \begin{pmatrix} \sin^2 \beta \cos^2 \beta - \cos^2 \beta \sin^2 \beta & \sin^2 \beta \cos \beta \sin \beta - \sin^2 \beta \cos \beta \sin \beta \\ -\cos^2 \beta \cos \beta \sin \beta + \cos^2 \beta \cos \beta \sin \beta & -\cos^2 \beta \sin^2 \beta + \cos^2 \beta \sin^2 \beta \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = BA \text{ as well. } \square \end{aligned}$$