Solving a system of linear equations

v. 1.24

PROBLEM

Solve the following system of 4 linear equations in 3 unknowns:

$$\begin{array}{rrrl} 4\,x_1 & +1\,x_2 & +0\,x_3 & = 0 \\ 0\,x_1 & -1\,x_2 & +2\,x_3 & = 0 \\ -2\,x_1 & +2\,x_2 & +1\,x_3 & = 0 \\ -2\,x_1 & +3\,x_2 & +4\,x_3 & = 0 \end{array}$$

SOLUTION

- Step 1: Transform the augmented matrix to the reduced row echelon form
- Step 2: Interpret the reduced row echelon form

Step 1: Transform the augmented matrix to the reduced row echelon form (Hide details)

Row Operation 1:

$$\begin{bmatrix} 4 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ -2 & 2 & 1 & 0 \\ -2 & 3 & 4 & 0 \end{bmatrix}$$

multiply the 1st row by **1/4**

$$\begin{bmatrix} 1 & \frac{1}{4} & 0 & 0 \\ 0 & -1 & 2 & 0 \\ -2 & 2 & 1 & 0 \\ -2 & 3 & 4 & 0 \end{bmatrix}$$

Row Operation 2:

$$\begin{bmatrix} 1 & \frac{1}{4} & 0 & 0 \\ 0 & -1 & 2 & 0 \\ -2 & 2 & 1 & 0 \\ -2 & 3 & 4 & 0 \end{bmatrix}$$

add **2** times the 1st row to the 3rd row

$$\begin{bmatrix} 1 & \frac{1}{4} & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & \frac{5}{2} & 1 & 0 \\ -2 & 3 & 4 & 0 \end{bmatrix}$$

1

1 1 0 0

Row Operation 3:

$$\begin{bmatrix} 1 & \frac{1}{4} & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & \frac{5}{2} & 1 & 0 \\ -2 & 3 & 4 & 0 \end{bmatrix}$$

add **2** times the 1st row to the 4th row

$$\begin{bmatrix} 1 & \frac{1}{4} & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & \frac{5}{2} & 1 & 0 \\ 0 & \frac{7}{2} & 4 & 0 \end{bmatrix}$$

Row Operation 4:

$$\begin{bmatrix} 1 & \frac{1}{4} & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & \frac{5}{2} & 1 & 0 \\ 0 & \frac{7}{2} & 4 & 0 \end{bmatrix}$$

multiply the 2nd row by **-1**

$$\begin{bmatrix} 1 & \frac{1}{4} & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & \frac{5}{2} & 1 & 0 \\ 0 & \frac{7}{2} & 4 & 0 \end{bmatrix}$$

Row Operation 5:

$$\begin{bmatrix} 1 & \frac{1}{4} & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & \frac{5}{2} & 1 & 0 \\ 0 & \frac{7}{2} & 4 & 0 \end{bmatrix}$$

add **-5/2** times the 2nd row to the 3rd row

$$\begin{bmatrix} 1 & \frac{1}{4} & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & \frac{7}{2} & 4 & 0 \end{bmatrix}$$

Row Operation 6:

$$\begin{bmatrix} 1 & \frac{1}{4} & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & \frac{7}{2} & 4 & 0 \end{bmatrix}$$

add **-7/2** times the 2nd row to the 4th row

$$\begin{bmatrix} 1 & \frac{1}{4} & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 11 & 0 \end{bmatrix}$$

Row Operation 7:

$$\begin{bmatrix} 1 & \frac{1}{4} & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 11 & 0 \end{bmatrix}$$

multiply the 3rd row by **1/6**

$$\begin{bmatrix} 1 & \frac{1}{4} & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 11 & 0 \end{bmatrix}$$

Row Operation 8:

$$\begin{bmatrix} 1 & \frac{1}{4} & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 11 & 0 \end{bmatrix}$$

add **-11** times the 3rd row to the 4th row

$$\begin{bmatrix} 1 & \frac{1}{4} & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Row Operation 9:

$$\begin{bmatrix} 1 & \frac{1}{4} & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

add **2** times the 3rd row to the 2nd row

$$\begin{bmatrix} 1 & \frac{1}{4} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Row Operation 10:

$$\begin{bmatrix} 1 & \frac{1}{4} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

add **-1/4** times the 2nd row to the 1st row

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 2: Interpret the reduced row echelon form

The reduced row echelon form of the augmented matrix is

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

which corresponds to the system

$$\begin{array}{rrrcl} 1\,x_1 & & & = & 0 \\ & 1\,x_2 & & = & 0 \\ & & 1\,x_3 & = & 0 \\ & & 0 & = & 0 \end{array}$$

No equation of this system has a form *zero = nonzero*; Therefore, the system is **consistent**.

The leading entries in the matrix have been highlighted in yellow.

A leading entry on the (i,j) position indicates that the j-th unknown will be determined using the i-th equation.

Since every column in the coefficient part of the matrix has a leading entry that means our system has a **unique solution**:

$x_1 = 0$

$x_2 = 0$

$x_3 = 0$

This concludes the solution of the problem. Do you want to

- solve another problem of the same type, or
- go to the main Toolkit page?