Practice Question Lecture # 35 to 45

Question

Find the dominant Eigen pair (i.e. the Eigen value and Eigen vector) by using the Power Method for the following matrix.

$$A = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Question

Perform next iteration for power method, where $Ax_o = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, where A=$\begin{bmatrix} 3 & 1 \\ 2 & 7 \end{bmatrix}$

Questions

Perform next iteration for power method, where $Ax_1 = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$, where A=$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$

Question

Check whether the matrix $\begin{bmatrix} \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ has orthonormal columns or not?

Question

Determine whether the vectors $\boldsymbol{y} = \begin{bmatrix} -2 \\ -3 \\ 4 \\ 1 \end{bmatrix}, \boldsymbol{z} = \begin{bmatrix} 7 \\ -2 \\ 1 \\ 4 \end{bmatrix}$ are orthogonal.

Question

Find the distance between $x = \begin{bmatrix} 7 \\ -3 \end{bmatrix} and\ y = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$.

Question:

Let $u = \begin{bmatrix} 3 \\ -4 \\ -2 \end{bmatrix}, v = \begin{bmatrix} 2 \\ -5 \\ 7 \end{bmatrix}$ .Compute and compare $u.v, \|u\|^2, \|v\|^2\ and\ \|u+v\|^2$.

Question:

Let $u = \begin{bmatrix} -3 \\ 4 \end{bmatrix}, v = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, w = \begin{bmatrix} 4 \\ -4 \\ -2 \end{bmatrix}$.

Find

$(a)\ \frac{v.u}{u.u}$

$(b)\ \|w\|$

(c) $(\frac{u.v}{v.v})v$

Question:

Express the vector v in terms of the orthogonal basis B = $\{u_1, u_2, u_3\}$, where

$$v = \begin{bmatrix} -2 \\ 3 \\ 5 \\ -1 \end{bmatrix}, u_1 = \begin{bmatrix} 2 \\ 1 \\ 3 \\ -1 \end{bmatrix}, u_2 = \begin{bmatrix} -3 \\ -1 \\ -1 \\ 0 \end{bmatrix}, u_3 = \begin{bmatrix} -3 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

Question:

Determine whether the set S= $\{u_1, u_2, u_3\}$ is an orthogonal set?

Where $u_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, $u_3 = \begin{bmatrix} -5 \\ -2 \\ 1 \end{bmatrix}$

Question

Compute the orthogonal projection of $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ onto the line through $\begin{bmatrix} 2 \\ -5 \end{bmatrix}$ and the origin.

Question:

Let $y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ *and* $u = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$. Compute the distance from y to the line through u and the origin.

Question:

Find the orthogonal projection of **y** onto $Span\{u_1, u_2\}$.

$$y = \begin{bmatrix} -8 \\ -5 \\ 4 \end{bmatrix}, u_1 = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}, u_2 = \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix}$$

Question:

Find a least square solution for the system ***Ax = b***

Where $A = \begin{bmatrix} 3 & 2 \\ 1 & 0 \\ 4 & 3 \end{bmatrix}, b = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$

Question

Apply the Gram-Schmidt process to transform the vectors $u_1 = (1, 0, 0), u_2 = (0,1,0), u_3 = (0,0,1)$ into an orthonormal basis.

Question

Let W = Span $\{x_1, x_2\}$, where $x_1 = \begin{bmatrix} 6 \\ 0 \\ -2 \end{bmatrix}, x_2 = \begin{bmatrix} -4 \\ 3 \\ -2 \end{bmatrix}$. Construct an orthogonal basis $\{v_1, v_2\}$ for W.

Question

Determine whether the vectors $u = (1, 2, -4, 3)$, $v = (-2, 1, -3, -4)$ are orthogonal with respect to Euclidean inner product.

Question

Let W be the subspace of R^2 spanned by $\begin{bmatrix} 4 \\ 6 \end{bmatrix}$. Find a unit vector that is a basis for W.

Question

Let $C[0, 2\pi]$ have the inner product $\int_0^{2\pi} f(t)g(t)dt$. Computer $\|\sin kt\|^2$ *for* $k > 0$.

Question

Write the Fourier coefficients a_k *and* b_k to the function $f(\mathrm{y}) = 2y$ on the interval $[0, 2\pi]$.