

Practice Questions for Lecture # 4

Question # 1

Identify if the following matrices are in echelon form or reduced echelon form or not in both forms. Also give the reason to justify your answer.

a.
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Echelon Form

b.
$$\begin{bmatrix} 6 & 2 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 \end{bmatrix}$$

Neither Echelon nor Reduced echelon forms because row with all zero elements is not the last row. All non-zero rows are above any rows of all zeros.

c.
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Reduced echelon form because:

The leading entry in each nonzero row is 1.

Each leading 1 is the only nonzero entry in its column

d.
$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$

Echelon form:

Each leading entry of a row is in a column to the right of the leading entry of the row above it.

All entries in a column below a leading entry are zero

e.
$$\begin{bmatrix} -1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Echelon form:

f.
$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

Echelon form:

Question # 2

For the given linear system $\begin{bmatrix} 2 & 3 & 5 & 7 \\ 0 & 2 & 3 & 9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, find the solution when free variable $z = 0$.

Solution:

Corresponding system of equation is

$$2x + 3y + 5z = 7 \quad \text{--- 1}$$

$$2y + 3z = 9 \quad \text{--- 2}$$

as given $z = 0$

put this value of z in equation 2 we get

$$2y + 0 = 9$$

$$y = 4.5$$

put the value of y and z in equation 1

$$2x + 3(4.5) + 0 = 7$$

$$2x + 13.5 = 7$$

$$2x = 7 - 13.5 = -6.5$$

$$x = 3.25$$

Question # 3

Apply elementary row operations to transform the following matrix into the reduced echelon form:

$$A = \begin{bmatrix} 3 & 6 & 9 \\ 1 & 3 & 3 \\ 4 & 8 & 5 \end{bmatrix}$$

Solution:

$$A = \begin{bmatrix} 3 & 6 & 9 \\ 1 & 3 & 3 \\ 4 & 8 & 5 \end{bmatrix}$$

$1/3R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 3 \\ 4 & 8 & 5 \end{bmatrix}$$

$R_2 - R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 4 & 8 & 5 \end{bmatrix}$$

$R_4 - 4R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & -7 \end{bmatrix}$$

$R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & -7 \end{bmatrix}$$

$-1/7R_3$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_1 - 3R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question # 4

Apply elementary row operations to transform the following matrix into the echelon form:

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 2 & 2 \\ 4 & -1 & -4 \end{bmatrix}$$

Solution:

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 2 & 2 \\ 4 & -1 & -4 \end{bmatrix}$$

$$R_2 - R_1, R_3 - 4R_1$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & -1 \\ 0 & -13 & -12 \end{bmatrix}$$

$$-R_2$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 1 \\ 0 & -13 & -12 \end{bmatrix}$$

$$R_3 + 13R_2$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Question # 5

Apply elementary row operations to transform the following matrix into

- Echelon form:
- Reduced Echelon form

$$A = \begin{bmatrix} 1 & -3 & -2 & 1 \\ -6 & 2 & 5 & 3 \\ 2 & 1 & 1 & -3 \end{bmatrix}$$

Solution:

$$A = \begin{bmatrix} 1 & -3 & -2 & 1 \\ -6 & 2 & 5 & 3 \\ 2 & 1 & 1 & -3 \end{bmatrix}$$

add 6 times the 1st row to the 2nd row

add -2 times the 1st row to the 3rd row

multiply the 2nd row by $-1/16$

add -7 times the 2nd row to the 3rd row

multiply the 3rd row by $16/31$

$$\begin{bmatrix} 1 & -3 & -2 & 1 \\ 0 & 1 & 7/16 & -9/16 \\ 0 & 0 & 1 & -17/31 \end{bmatrix}$$

The matrix is in echelon form

To convert this in reduced echelon form

add $-7/16$ times the 3rd row to the 2nd row

add 2 times the 3rd row to the 1st row

add 3 times the 2nd row to the 1st row

$$\begin{bmatrix} 1 & 0 & 0 & -33/31 \\ 0 & 1 & 0 & -10/31 \\ 0 & 0 & 1 & -17/31 \end{bmatrix}$$

Question # 6

Find the general solution of a linear system whose augmented matrix is $\begin{pmatrix} 1 & 1 & 3 \\ 3 & 2 & 1 \end{pmatrix}$

Solution:

Corresponding system of linear equation

$$x + y = 3$$

$$3x + 2y = 1$$

Multiply equation 1 with 3 and subtract from second

$$3x + 3y = 9$$

$$3x + 2y = 1$$

$$\begin{array}{r} - \quad - \quad - \\ \hline \end{array}$$

$$0 + y = 8$$

put the value of y in first equation

$$x + 8 = 3$$

$$x = 3 - 8 = -5$$

Question # 7

Find the general solution of a linear system whose augmented matrix is $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

Corresponding system of linear equation

$$x + 0y = 0$$

$$0x + y = 0$$

$$x = 0$$

$$y = 0$$