Orthogonal Basis

Theorem:

Given a basis $\{x_1, x_2, \dots, x_p\}$ for a subspace W of R^n. Then

$$v_1 = x_1, v_2 = x_2 - P$$

$$v_3 = x_3 - P$$

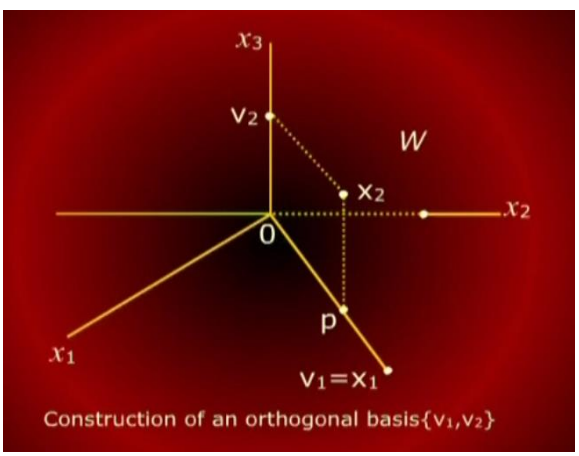
$$\dots \dots \dots \dots \dots$$

$$v_p = x_p - P$$

Then $\{v_1, v_2, \dots, v_p\}$ is an orthogonal basis for W. In addition

$$Span\{v_1, v_2, \dots, v_k\} = Span\{x_1, x_2, \dots, x_k\} for\ 1 \leq k \leq p$$

Example # 1



Construction of an orthogonal basis{v1,v2}

This is the graphical representation of example # 1 on page # 485. In this example we have two vectors $x_1\ and\ x_2$. W is the vector space and P is the projection of x_2 on x_1. Next the component of x_2 orthogonal to x_1 is

$$x_2 = x_1 - P$$

Which is in W because it formed by x_1 and multiples of $x_1 x_2$.