$\because$ it is given that $T(e_1) = T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = y_1 = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ and

$T(e_2) = T\begin{pmatrix} 0 \\ 1 \end{pmatrix} = y_2 = \begin{pmatrix} -2 \\ 7 \end{pmatrix}$

$\Longrightarrow$ the transformation matrix say $A = \begin{pmatrix} T(e_1) & T(e_2) \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ -5 & 7 \end{pmatrix}$

Now the image of $\begin{pmatrix} 7 \\ 6 \end{pmatrix} = A\begin{pmatrix} 7 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ -5 & 7 \end{pmatrix}\begin{pmatrix} 7 \\ 6 \end{pmatrix} = \begin{pmatrix} 9 \\ 7 \end{pmatrix}$

And the image of $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = A\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ -5 & 7 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3x_1 - 2x_2 \\ 7x_2 - 5x_1 \end{pmatrix}$

Alternatively, you can find this as;

$T\begin{pmatrix} 7 \\ 6 \end{pmatrix} = T\begin{pmatrix} 7+0 \\ 0+6 \end{pmatrix} = T\left[\begin{pmatrix} 7+0 \\ 0+6 \end{pmatrix}\right] = T\left[\begin{pmatrix} 7 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 6 \end{pmatrix}\right]$

T is linear and satisfy $T(x+y) = Tx + Ty$

$\therefore T\begin{pmatrix} 7 \\ 6 \end{pmatrix} = T\begin{pmatrix} 7 \\ 0 \end{pmatrix} + T\begin{pmatrix} 0 \\ 6 \end{pmatrix} = T\left(7\begin{matrix} 1 \\ 0 \end{matrix}\right) + T\left(6\begin{matrix} 0 \\ 1 \end{matrix}\right)$

$\because T$ is linear and satisfy $T(\alpha x) = \alpha T(x)$

$\Longrightarrow T\begin{pmatrix} 7 \\ 6 \end{pmatrix} = 7T\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 6T\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\because$it is given that $T(e_1) = T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = y_1 = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ and

$T(e_2) = T\begin{pmatrix} 0 \\ 1 \end{pmatrix} = y_2 = \begin{pmatrix} -2 \\ 7 \end{pmatrix}$

$\therefore T\begin{pmatrix} 7 \\ 6 \end{pmatrix} = 7\begin{pmatrix} 3 \\ -5 \end{pmatrix} + 6\begin{pmatrix} -2 \\ 7 \end{pmatrix} = \begin{pmatrix} 9 \\ 7 \end{pmatrix}$