Since we know that the *vectors say* $\overrightarrow{v}_1, \overrightarrow{v}_2, \cdots, \overrightarrow{v}_n$ *in* $\mathbb{R}^n$ *are linearly independent*$\Longleftrightarrow$*whenever for the scalars* $\alpha_1, \alpha_2, \cdots, \alpha_n$,*the equation:*

$\alpha_1 \overrightarrow{v}_1 + \alpha_2 \overrightarrow{v}_2 + \cdots + \alpha_n \overrightarrow{v}_n = \overrightarrow{0}$ *has the trivial solution(zero) and linearly dependent* $\Longleftrightarrow$ *that equation has non-trivial solution(least one* $non - zero$*).*

Now we apply the above on the following questions:

**Question 16**:

If $\overrightarrow{v}_1, \ldots, \overrightarrow{v}_4$ are in $\mathbb{R}^4$, and $\overrightarrow{v}_3 = 2\overrightarrow{v}_1 + \overrightarrow{v}_2$, then $\overrightarrow{v}_1, \cdots, \overrightarrow{v}_4$ is linearly dependent.

**Solution:**

Given that

$\overrightarrow{v}_3 = 2\overrightarrow{v}_1 + \overrightarrow{v}_2$

$\Longrightarrow -2\overrightarrow{v}_1 - \overrightarrow{v}_2 + \overrightarrow{v}_3 = \overrightarrow{0}$

$\Longrightarrow (-2)\,\overrightarrow{v}_1 + (-1)\,\overrightarrow{v}_2 + (1)\,\overrightarrow{v}_3 + 0\overrightarrow{v}_4 = \overrightarrow{0}$

$\Longrightarrow \alpha_1 = -2, \alpha_2 = -1, \alpha_3 = 1$ and $\alpha_4 = 0$ i.e. not all the scalars are zeros$\Longrightarrow$ the given vectors are Linearly Dependent.

**Question 17:**

If $\overrightarrow{v}_1$ and $\overrightarrow{v}_2$ are in $\mathbb{R}^4$, and $\overrightarrow{v}_1$ is not a scalar multiple of $\overrightarrow{v}_2$, then $\{\overrightarrow{v}_1, \overrightarrow{v}_2\}$ is linearly independent.

**Solution:**

If $\overrightarrow{v}_1$ is a scalar multiple of $\overrightarrow{v}_2$, then $\exists$ a scalar say $\alpha \neq 0 \in \mathbb{R}$ such that

$\overrightarrow{v}_1 = \alpha\overrightarrow{v}_2 \Longrightarrow \overrightarrow{v}_1 - \alpha\overrightarrow{v}_2 = \overrightarrow{0} \Longrightarrow (1)\overrightarrow{v}_1 + \alpha\overrightarrow{v}_2 = \overrightarrow{0} \Longrightarrow \alpha_1 = 1, \alpha_2 = \alpha \neq 0$ i.e not all are zeros$\Longrightarrow$ the $\overrightarrow{v}_1$ and $\overrightarrow{v}_2$ are linearly dependent.

But it is given that $\overrightarrow{v}_1$ is not a scalar multiple of $\overrightarrow{v}_2 \Longrightarrow$ the given vectors are linearly Independent.

**Question 18**:.

If $\overrightarrow{v}_1, \ldots, \overrightarrow{v}_4$ are in $\mathbb{R}^4$, and $\{\overrightarrow{v}_1, \overrightarrow{v}_2, \overrightarrow{v}_3\}$ is linearly dependent, then$\{\overrightarrow{v}_1, \overrightarrow{v}_2, \overrightarrow{v}_3, \overrightarrow{v}_4\}$ is also linearly dependent.

**Solution**:

$\{\overrightarrow{v}_1, \overrightarrow{v}_2, \overrightarrow{v}_3\}$in $\mathbb{R}^4$ are linearly dependent

$\therefore \alpha_1\overrightarrow{v}_1 + \alpha_2\overrightarrow{v}_2 + \alpha_3\overrightarrow{v}_3 = \overrightarrow{0} \Longrightarrow$ not all the scalars are zeros.

$\Longrightarrow \alpha_1\overrightarrow{v}_1 + \alpha_2\overrightarrow{v}_2 + \alpha_3\overrightarrow{v}_3 + 0\overrightarrow{v}_4 = \overrightarrow{0}$

$\Longrightarrow \alpha_1\overrightarrow{v}_1 + \alpha_2\overrightarrow{v}_2 + \alpha_3\overrightarrow{v}_3 + 0\overrightarrow{v}_4 = \overrightarrow{0}$ again all the scalars are not zeros.

$\Longrightarrow \{\overrightarrow{v}_1, \overrightarrow{v}_2, \overrightarrow{v}_3, \overrightarrow{v}_4\}$ is linearly dependent.