Question:

Write the solution set of the given homogeneous system in parametric vector form.

$x+2y-7z=0$
$-2x-3y+9z=0$
$0x-2y+10z=0$

Solution:

For the given system: $\begin{matrix} x+2y-7z=0 \\ -2x-3y+9z=0 \\ 0x-2y+10z=0 \end{matrix}$, the Corresponding augmented matrix: $\begin{pmatrix} 1 & 2 & -7 & 0 \\ -2 & -3 & 9 & 0 \\ 0 & -2 & 10 & 0 \end{pmatrix}$.Now we reduce this into reduce Echelon form as follows:

$$\begin{pmatrix} 1 & 2 & -7 & 0 \\ -2 & -3 & 9 & 0 \\ 0 & -2 & 10 & 0 \end{pmatrix}$$

By $R_2' \to R_2 + 2R_1$

$$\sim \begin{pmatrix} 1 & 2 & -7 & 0 \\ -2+2(1) & -3+2(2) & 9+2(-7) & 0 \\ 0 & -2 & 10 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 & -7 & 0 \\ 0 & 1 & -5 & 0 \\ 0 & -2 & 10 & 0 \end{pmatrix}$$

By $R_1' \to R_1 - 2R_2$, $R_3' \to R_3 + 2R_2$

$$\sim \begin{pmatrix} 1-2(0) & 2-2(1) & -7-2(-5) & 0-2(0) \\ 0 & 1 & -5 & 0 \\ 0+2(0) & -2+2(1) & 10+2(-5) & 0+2(0) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$ i.e. the reduced Echelon form.

Now last row$\Longrightarrow$ $0z = 0$,which is true $\forall z \in \mathbb{R}$,as every real number on multiplying with *zero* always gives a *zero*.

2nd row $\Longrightarrow$ $y-5z=0 \Longrightarrow y=5z$,which means that for infinite many values of $z \in \mathbb{R}, \exists$ corresponding infinite many values of y.

1st row $\Longrightarrow x-3z=0 \Longrightarrow x=3z$,which again means that for infinite many values of $z \in \mathbb{R}, \exists$ corresponding infinite many values of x.

In all three cases, we see that choice of z is arbitrary, so z can be taken as free variable or parameter. Now expressing this solution in vector form:

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3z \\ 5z \\ z \end{pmatrix} = z \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix}$ i.e. the required vector solution in parametric form. Hence the given system has the solution space generated by the vector $\begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix}$.