

Question:

Do the columns of the matrix $A = \begin{pmatrix} 1 & 3 & -2 & -2 \\ 0 & 1 & -1 & 5 \\ -1 & -2 & 1 & 7 \\ 1 & 1 & 0 & -6 \end{pmatrix}$ span \mathbb{R}^4 ?

Solution:

The columns of matrix A will span $\mathbb{R}^4 \iff \det A \neq 0$ i.e. it is non-singular.

Now for $\det A = \begin{vmatrix} 1 & 3 & -2 & -2 \\ 0 & 1 & -1 & 5 \\ -1 & -2 & 1 & 7 \\ 1 & 1 & 0 & -6 \end{vmatrix}$, we apply the elementary row operations to

have maximum zeros in Pivot columns.

$$\text{By } R'_3 \rightarrow R_3 + R_1, R'_4 \rightarrow R_4 - R_1$$

$$= \begin{vmatrix} 1 & 3 & -2 & -2 \\ 0 & 1 & -1 & 5 \\ -1+1 & -2+3 & 1+(-2) & 7+(-2) \\ 1-1 & 1-3 & 0-(-2) & -6-(-2) \end{vmatrix} = \begin{vmatrix} 1 & 3 & -2 & -2 \\ 0 & 1 & -1 & 5 \\ 0 & 1 & -1 & 5 \\ 0 & -2 & 2 & -4 \end{vmatrix}$$

$$\text{By } R'_3 \rightarrow R_3 - R_2, R'_4 \rightarrow R_4 + 2R_2$$

$$= \begin{vmatrix} 1 & 3 & -2 & -2 \\ 0 & 1 & -1 & 5 \\ 0 & 1-1 & -1-(-1) & 5-5 \\ 0 & -2+2(1) & 2+2(-1) & -4+2(5) \end{vmatrix} = \begin{vmatrix} 1 & 3 & -2 & -2 \\ 0 & 1 & -1 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 \end{vmatrix} \text{ which}$$

contains a zero row.

$$\implies \det A = 0$$

\therefore columns of matrix A do not span \mathbb{R}^4 .