Question:

It can be shown that $\begin{pmatrix} 4 & 1 & 2 \\ -2 & 0 & 8 \\ 3 & 5 & -6 \end{pmatrix} \begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 18 \\ 5 \end{pmatrix}$. Use this fact(and no row operations) to find scalars c1, c2, c3 such that $\begin{pmatrix} 4 \\ 18 \\ 5 \end{pmatrix} = c_1 \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix} + c_3 \begin{pmatrix} 2 \\ 8 \\ -6 \end{pmatrix}$.

Solution:

Since it is given:

$\begin{pmatrix} 4 & 1 & 2 \\ -2 & 0 & 8 \\ 3 & 5 & -6 \end{pmatrix} \begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 18 \\ 5 \end{pmatrix}$.So we solve LHS by usual multiplication.

$\Longrightarrow \begin{pmatrix} 4(-1) + 1(4) + 2(2) \\ -2(-1) + 0(4) + 8(2) \\ 3(-1) + 5(4) + (-6)(2) \end{pmatrix} = \begin{pmatrix} 4 \\ 18 \\ 5 \end{pmatrix}$

$\Longrightarrow \begin{pmatrix} 4(-1) \\ -2(-1) \\ 3(-1) \end{pmatrix} + \begin{pmatrix} 1(4) \\ 0(4) \\ 5(4) \end{pmatrix} + \begin{pmatrix} 2(2) \\ 8(2) \\ -6(2) \end{pmatrix} = \begin{pmatrix} 4 \\ 18 \\ 5 \end{pmatrix}$

$\Longrightarrow (-1) \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} + 4 \begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix} + 2 \begin{pmatrix} 2 \\ 8 \\ -6 \end{pmatrix} = \begin{pmatrix} 4 \\ 18 \\ 5 \end{pmatrix}$

$\Longrightarrow \begin{pmatrix} 4 \\ 18 \\ 5 \end{pmatrix} = (-1) \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} + 4 \begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix} + 2 \begin{pmatrix} 2 \\ 8 \\ -6 \end{pmatrix}$

Now it can be comapraed with the required expression and can be observed that:

$c_1 = -1, c_2 = 4$ and $c_3 = 2$.