

**Question:**

Let  $\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$ ,  $\vec{v}_2 = \begin{pmatrix} -2 \\ 1 \\ 7 \end{pmatrix}$  and  $\vec{y} = \begin{pmatrix} h \\ -3 \\ -5 \end{pmatrix}$ . For what value(s)

of  $h$ , is  $\vec{y}$  in the plane generated by  $\vec{v}_1$  and  $\vec{v}_2$ ?

**Solution:**

$\therefore$  we know if a vector  $\vec{y}$  is in the plane generated by  $\vec{v}_1$  and  $\vec{v}_2$   
then  $\implies \vec{y}$  can be written as a linear combination of  $\vec{v}_1$  and  $\vec{v}_2$ ,

then  $\implies$  vector equation:  $\vec{y} = x \vec{v}_1 + y \vec{v}_2$  has the solution, where  $x$  and  $y$  are unknowns.

$$\begin{aligned} \therefore \vec{y} &= x \vec{v}_1 + y \vec{v}_2 \\ \implies \begin{pmatrix} h \\ -3 \\ -5 \end{pmatrix} &= x \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} + y \begin{pmatrix} -2 \\ 1 \\ 7 \end{pmatrix} \end{aligned}$$

$$\implies \begin{pmatrix} h \\ -3 \\ -5 \end{pmatrix} = \begin{pmatrix} x \\ 0 \\ -2x \end{pmatrix} + \begin{pmatrix} -2y \\ y \\ 7y \end{pmatrix}$$

$$\implies \begin{pmatrix} h \\ -3 \\ -5 \end{pmatrix} = \begin{pmatrix} x - 2y \\ y \\ 7y - 2x \end{pmatrix}, \text{ the corresponding system of equation: } \begin{aligned} x - 2y &= h \\ y &= -3 \\ -2x + 7y &= -5 \end{aligned}$$

From 2nd equation, put  $y = -3$  in 3rd equation:  $-2x + 7y = -5 \implies -2x +$

$$7(-3) = -5 \implies x = -8$$

$\therefore$  1st equation  $\implies (-8) - 2(-3) = h \implies h = -2$  is the required value.