

Prove that $\vec{u} + \vec{v} = \vec{v} + \vec{u}$ for any \vec{u} and \vec{v} in \mathbb{R}^n .

Proof:

Let $u, v \in \mathbb{R}^n$.

$\implies \vec{u} = (x_1, x_2, \dots, x_n), \vec{v} = (y_1, y_2, \dots, y_n)$, where all x and y 's are members from \mathbb{R} .

Here we are to prove the commutative law in \mathbb{R}^n :

$$\vec{u} + \vec{v} = \vec{v} + \vec{u},$$

while we know that commutative law under addition " + " holds in \mathbb{R} i.e.

$$x + y = y + x \quad \text{--- (1)} \quad \forall x, y \in \mathbb{R}$$

Now the addition in \mathbb{R}^n is defined as;

$$\vec{u} + \vec{v} = (x_1, x_2, \dots, x_n) + (y_1, y_2, \dots, y_n) = (x_1 + y_1, x_2 + y_2, \dots, x_n + y_n)$$

$$\implies \vec{u} + \vec{v} = (y_1 + x_1, y_2 + x_2, \dots, y_n + x_n) \quad \because \text{by using (1).}$$

$$\implies \vec{u} + \vec{v} = (y_1 + x_1, y_2 + x_2, \dots, y_n + x_n) = (y_1, y_2, \dots, y_n) + (x_1, x_2, \dots, x_n)$$

$$\implies \vec{u} + \vec{v} = \vec{v} + \vec{u}. \quad \square$$