The mapping $T : P_2 \to P_2$ defined by $T(a_0 + a_1 t + a_2 t^2) = a_1 + 2a_2 t$ → Here the given transformation is the derivative of a polynomial.
is a linear transformation.

i.e $T(a_0+a_1t+a_2t^2) = \frac{d}{dt}(a_0+a_1t+a_2t^2) = a_1+2a_2t$

(a) Find the B-matrix for T, when B is the basis $\{1, t, t^2\}$.
(b) Verify that $[T(p)]_B = [T]_B[p]_B$ for each p in P_2.

$\therefore T(1) = 0$ $\because$ the derivative of constant is zero.

$T(t) = 1$ $\because$ derivative of 't' w.r.t 't' is '1'

$T(t^2) = 2t$ $\because \frac{d}{dt}t^n = nt^{n-1}$

Solution

(a) We have to find the B-matrix of $T : V \to V$ satisfies

$$[T(x)]_B = [T]_B[x]_B, \quad \text{for all } x \text{ in } V$$

Since $T(a_0 + a_1 t + a_2 t^2) = a_1 + 2a_2 t$ therefore

$$T(1) = 0,\ T(t) = 1,\ T(t^2) = 2t$$

$$[T(1)]_B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, [T(t)]_B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, [T(t^2)]_B = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$[T]_B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Now note the columns of basis will be the coefficients of polynomial after taking the derivative.

$\therefore T(1) = 0 = 0 + 0t + 0t^2 \Rightarrow T(1) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$T(t) = 1 = 1 + 0t + 0t^2 \Rightarrow T(t) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$T(t^2) = 2t = 0 + 2t + 0t^2 \Rightarrow T(t^2) = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$