

It is incorrect to say that the system has no solution. Here after reducing the Augmented matrix into Echelon form, the associated reduced system of equations are;

$$x_1 + 5x_2 + 2x_3 = -6 \text{ --- (1)}$$

$$4x_2 - 7x_3 = 2 \text{ --- (2)}$$

$$5x_3 = 0 \text{ --- (3)}$$

Here the last equation (3) implies;

$$5x_3 = 0 \implies x_3 = \frac{0}{5} = 0$$

$$\therefore \boxed{x_3 = 0} \text{ --- put in (2)}$$

$$\therefore (2) \implies 4x_2 - 7(0) = 2$$

$$\implies 4x_2 = 2 \implies x_2 = \frac{2}{4} = \frac{1}{2}$$

$$\therefore \boxed{x_2 = \frac{1}{2}} \text{ --- put in (1)}$$

$$\therefore (1) \implies x_1 + 5\left(\frac{1}{2}\right) + 2(0) = -6$$

$$\implies x_1 + \frac{5}{2} = -6$$

$$\implies x_1 = -\frac{5}{2} - 6$$

$$\implies \boxed{x_1 = -\frac{17}{2}}$$