

The Eigen space corresponding to eigen value  $\lambda = 5$  is obtained by solving  $Ax = \lambda x$

$$\begin{aligned} &\implies (A - \lambda I)x = 0 \\ &\implies (A - 5I)x = 0 \\ &\implies \left\{ \begin{pmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & h & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix} - 5 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right\} x = 0 \\ &\implies \begin{pmatrix} 0 & -2 & 6 & -1 \\ 0 & -2 & h & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & -4 \end{pmatrix} x = 0 \end{aligned}$$

$\therefore$  the associated augmented matrix is;

$$\begin{pmatrix} 0 & -2 & 6 & -1 & 0 \\ 0 & -2 & h & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & -4 & 0 \end{pmatrix}$$

By  $R_2' \rightarrow R_2 - R_1, R_4' \rightarrow R_3 + R_4$

$$\begin{pmatrix} 0 & -2 & 6 & -1 & 0 \\ 0 & 0 & h-6 & 1 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & -4 & 0 \end{pmatrix}$$

And on further elementary row operations, we get

$$\begin{pmatrix} 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & h-6 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Since the given Eigen subspace is of two dimension, so there will be two free variables and two non zero rows, which can only be possible if the 2nd row is also zero, which necessarily means

$$h - 6 = 0 \text{ or } \boxed{h = 6}$$