Given matrix is $A = \begin{pmatrix} 4 & 0 & 0 \\ 5 & 3 & 2 \\ -2 & 0 & 2 \end{pmatrix}$

Taking

$$A - \lambda I = \begin{pmatrix} 4 & 0 & 0 \\ 5 & 3 & 2 \\ -2 & 0 & 2 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 4-\lambda & 0 & 0 \\ 5 & 3-\lambda & 2 \\ -2 & 0 & 2-\lambda \end{pmatrix}$$

$\implies |A - \lambda I| = -\lambda^3 + 9\lambda^2 - 26\lambda + 24$ i.e. characteristic polynomial.

For the Eigen values, put $|A - \lambda I| = 0$,

$\implies -\lambda^3 + 9\lambda^2 - 26\lambda + 24 = 0$

Factorizing, by synthetic division or by inspection,

$(\lambda - 3)(\lambda - 4)(\lambda - 2) = 0$

$\therefore$ the required eigen values are:

$\lambda = 2, 3, 4$