Theorem 1 If a vector space V has a basis $B = \{b_1, ..., b_n\}$, then any set in V co
more than n vectors must be linearly dependent.

Proof:

Let $B' = \{b1, b2, \cdots, b_n, b\} \subset V$ be a set having $n+1$ elements.
$\because \{b1, b2, \cdots, b_n\}$ is a basis for V
$\Longrightarrow$ each vector of V (eventually for $b \in B'$)can be written as a linear combination of vectors from $\{b1, b2, \cdots, b_n\}$
$\therefore$ for $b \in B', \exists\ \alpha_1, \alpha_2, \cdots, \alpha_n \in F$ such that

$$b = \alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_3$$

$\Longrightarrow \alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_3 + (-1)b = 0$
$\Longrightarrow -1 \neq 0$, even if all $\alpha_1 = \alpha_2 = \cdots = \alpha_n = 0$
$\Longrightarrow B' = \{b1, b2, \cdots, b_n, b\}$ is linearly dependent.