Let $V$ be the space spanned by $v_1 = \cos^2 x$, $v_2 = \sin^2 x, v_3 = \cos 2x$.
**(a)** Show that $S = \{v_1, v_2, v_3\}$ is not a basis for $V$ **(b)** Find a basis for $V$.

**Solution:**
**(a)**
$\because \cos 2x = \cos^2 x - \sin^2 x$ i.e. one vector can be expressed as a linear combination of others.
$\implies v_3 = v_1 - v_2 \implies v_3 + (-1)v_1 + 1v_2 = \overrightarrow{0} \implies$ the given vectors are linearly dependent
$\implies S = \{v_1, v_2, v_3\}$ is not a basis for $V$ .
**(b)**
$v_1 = \cos^2 x$, $v_2 = \sin^2 x$ are linearly independent as one cant be expressed as linear combination of other.
$\implies S^{'} = \{v_1, v_2\}$ is linearly independent and can serve as Basis for $V$.