Question:

Determine whether the solution space of the system $A\overrightarrow{x} = \overrightarrow{0}$ is a line through the origin, a plane through the origin, or the origin only. If it is a plane, find an equation for it; if it is a line, find parametric equations for it.

$A = \begin{pmatrix} 1 & -2 & 3 \\ -3 & 6 & 9 \\ -2 & 4 & -6 \end{pmatrix}$

Solution:

$\because$ the given matrix is of 3×3,and $\overrightarrow{x}$ is a column vector, then by the definition of matrix multiplication, $\overrightarrow{x}$ is of 3×1 order say $\overrightarrow{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

Now $A\overrightarrow{x} = \overrightarrow{0}$

$\Longrightarrow \begin{pmatrix} 1 & -2 & 3 \\ -3 & 6 & 9 \\ -2 & 4 & -6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\Longrightarrow \begin{pmatrix} x-2y+3z \\ 6y-3x+9z \\ 4y-2x-6z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\Longrightarrow \begin{matrix} x-2y+3z=0 \\ 6y-3x+9z=0 \\ 4y-2x-6z=0 \end{matrix}$ is the associated system of equations, whose augmented matrix is: $\begin{matrix} 1 & -2 & 3 & 0 \\ -3 & 6 & 9 & 0 \\ -2 & 4 & -6 & 0 \end{matrix}$. Now we reduce this into Echelon form:

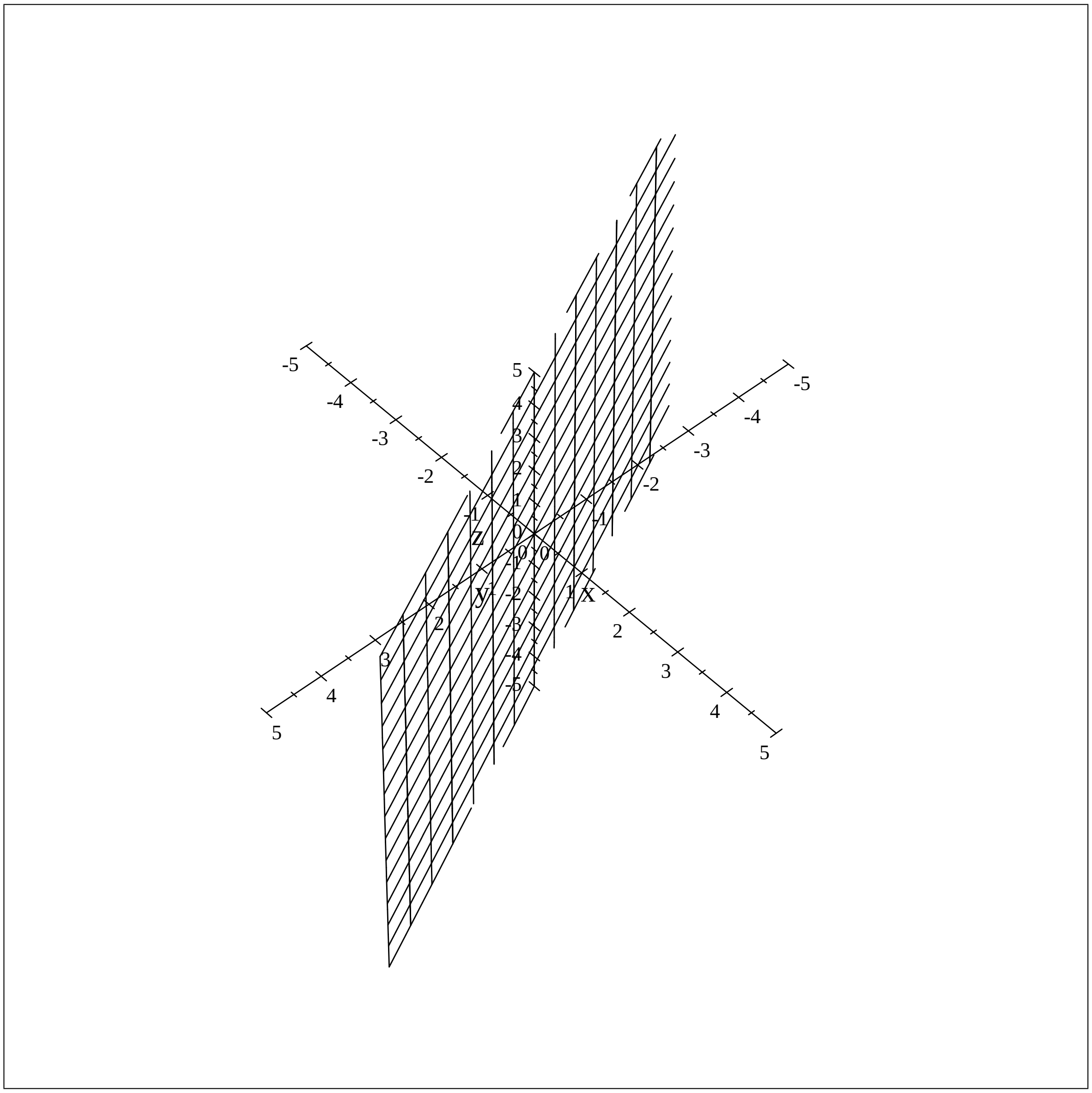
$\begin{pmatrix} 1 & -2 & 3 & 0 \\ -3 & 6 & 9 & 0 \\ -2 & 4 & -6 & 0 \end{pmatrix}$

By $R_2' \to R_2 + 3R_1$ and $R_3' \to R_3 + 3R_1$

$\sim \begin{pmatrix} 1 & -2 & 3 & 0 \\ -3+3(1) & 6+3(-2) & 9+3(3) & 0 \\ -2+2(1) & 4+2(-2) & -6+2(3) & 0 \end{pmatrix} = \begin{pmatrix} 1 & -2 & 3 & 0 \\ 0 & 0 & 18 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\therefore$last row $\Longrightarrow 0z = 0 \Longrightarrow z$ is arbitrary and 2nd row$\Longrightarrow 18z = 0 \Longrightarrow z = 0$. Hence, $z = \{z/z \in \mathbb{R}\} \cap \{0\} = \{0\}$

1st row$\Longrightarrow x - 2y + 3z = 0 \Longrightarrow x - 2y = 0$, which is the equation of plane bisecting the xy-plane.


x
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