$X=\begin{pmatrix} \frac{1}{2}\sin 2t-4\cos 2t \\ -3\sin 2t+5\cos 2t \end{pmatrix}$

Taking derivative on both sides with respect to t

$\frac{d}{dt}X=\frac{d}{dt}\begin{pmatrix} \frac{1}{2}\sin 2t-4\cos 2t \\ -3\sin 2t+5\cos 2t \end{pmatrix}$

$\because \left\{ \frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{d}{dt}x \\ \frac{d}{dt}y \end{pmatrix} \right.$

$\Longrightarrow \frac{d}{dt}X=\begin{pmatrix} \frac{d}{dt}\left(\frac{1}{2}\sin 2t-4\cos 2t\right) \\ \frac{d}{dt}\left(-3\sin 2t+5\cos 2t\right) \end{pmatrix}$

$\because \left\{ \frac{d}{dt}\left(x\pm y\right)^1=\frac{d}{dt}x\pm\frac{d}{dt}y \right.$

$\Longrightarrow \frac{d}{dt}X=\begin{pmatrix} \frac{d}{dt}\left(\frac{1}{2}\sin 2t\right)-\frac{d}{dt}\left(4\cos 2t\right) \\ \frac{d}{dt}\left(-3\sin 2t\right)+\frac{d}{dt}\left(5\cos 2t\right) \end{pmatrix}$

$\because \left\{ \frac{d}{dt}\left(\alpha f(t)\right)=\alpha\frac{d}{dt}f(t) \right.$

$\Longrightarrow \frac{d}{dt}X=\begin{pmatrix} \frac{1}{2}\frac{d}{dt}\sin 2t-4\frac{d}{dt}\cos 2t \\ -3\frac{d}{dt}\sin 2t+5\frac{d}{dt}\cos 2t \end{pmatrix}$

$\because \left\{ \begin{array}{c} \frac{d}{dt}\sin\alpha t=\alpha\cos\alpha t \\ \frac{d}{dt}\cos\alpha t=-\alpha\sin\alpha t \end{array} \right.$

$\Longrightarrow \frac{d}{dt}X=\begin{pmatrix} \frac{1}{2}\left(2\cos 2t\right)-4\left(-2\sin 2t\right) \\ -3\left(2\cos 2t\right)+5\left(-2\sin 2t\right) \end{pmatrix}$

$\Longrightarrow \boxed{\frac{d}{dt}X=\begin{pmatrix} \cos 2t+8\sin 2t \\ -6\cos 2t-10\sin 2t \end{pmatrix}}$