

$$\text{Show that } \begin{vmatrix} a_1 + b_1t & a_2 + b_2t & a_3 + b_3t \\ a_1t + b_1 & a_2t + b_2 & a_3t + b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = (1 - t^2) \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}.$$

Solution:

R.H.S

$$= \begin{vmatrix} a_1 + b_1t & a_2 + b_2t & a_3 + b_3t \\ a_1t + b_1 & a_2t + b_2 & a_3t + b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

By $R'_1 \rightarrow R_1 - tR_2$

$$= \begin{vmatrix} (a_1 + b_1t) - t(a_1t + b_1) & (a_2 + b_2t) - t(a_2t + b_2) & (a_3 + b_3t) - t(a_3t + b_3) \\ a_1t + b_1 & a_2t + b_2 & a_3t + b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} -a_1(t^2 - 1) & -a_2(t^2 - 1) & -a_3(t^2 - 1) \\ a_1t + b_1 & a_2t + b_2 & a_3t + b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

By $R'_1 \rightarrow (t^2 - 1) \left(\frac{1}{(t^2 - 1)} R_1 \right)$

$$= (t^2 - 1) \begin{vmatrix} \frac{-a_1(t^2 - 1)}{(t^2 - 1)} & \frac{-a_2(t^2 - 1)}{(t^2 - 1)} & \frac{-a_3(t^2 - 1)}{(t^2 - 1)} \\ a_1t + b_1 & a_2t + b_2 & a_3t + b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= (t^2 - 1) \begin{vmatrix} -a_1 & -a_2 & -a_3 \\ a_1t + b_1 & a_2t + b_2 & a_3t + b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

By $R'_1 \rightarrow (-1) R_1$

$$= (1 - t^2) \begin{vmatrix} a_1 & a_2 & a_3 \\ a_1t + b_1 & a_2t + b_2 & a_3t + b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

By $R'_2 \rightarrow R_2 - tR_1$

$$= (1 - t^2) \begin{vmatrix} a_1 & a_2 & a_3 \\ (a_1t + b_1) - a_1t & (a_2t + b_2) - a_2t & (a_3t + b_3) - a_3t \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= (1 - t^2) \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \text{RHS}$$