

In the given questions 1-3 of lecture-14, we are to find the formulas for X, Y and Z in terms of A, B and C provided that the given matrix equations are satisfied.

$$\begin{aligned} &\because \text{ here given that } \begin{pmatrix} A & B \\ C & O \end{pmatrix} \begin{pmatrix} I & O \\ X & Y \end{pmatrix} = \begin{pmatrix} O & I \\ Z & O \end{pmatrix} \\ \Rightarrow &\begin{pmatrix} AI + BX & AO + BY \\ CI + OX & CO + OY \end{pmatrix} = \begin{pmatrix} O & I \\ Z & O \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} A + BX & BY \\ C & O \end{pmatrix} = \begin{pmatrix} O & I \\ Z & O \end{pmatrix} \end{aligned}$$

Equating the corresponding entries on both sides;

$$\Rightarrow A + BX = O,$$

$$BY = I,$$

$$\boxed{C = Z}$$

and

$$O = O$$

Here $BY = I$

$\Rightarrow B$ is invertible

$$\therefore BB^{-1}Y = B^{-1}I$$

$$\Rightarrow IY = B^{-1}I$$

$$\Rightarrow \boxed{Y = B^{-1}}$$

Now from $A + BX = O$

$$\Rightarrow BX = -A$$

$$\Rightarrow B^{-1}BX = B^{-1}(-A)$$

$$\Rightarrow IX = -B^{-1}A$$

$$\Rightarrow \boxed{X = -B^{-1}A}$$