Question:

Assume that T is a linear transformation. Find the standard matrix of $T:\mathbb{R}^3\to\mathbb{R}^3$ projects each point (x_1,x_2,x_3) vertically onto the x_1x_2-plane (where $x_3=0$).

Solution:

$\because$ the standard basis vectors in $\mathbb{R}^3$are: $\begin{pmatrix}1\\0\\0\end{pmatrix}$, $\begin{pmatrix}0\\1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\0\\1\end{pmatrix}$ and given that $T:\mathbb{R}^3\to\mathbb{R}^3$ projects each point (x_1,x_2,x_3) vertically onto the x_1x_2-plane (where $x_3=0$)

$\Longrightarrow T\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}x_1\\x_2\\0\end{pmatrix}$ $\Longrightarrow$ for the standard basis vectors, we have:

$T\begin{pmatrix}1\\0\\0\end{pmatrix}=\begin{pmatrix}1\\0\\0\end{pmatrix}, T\begin{pmatrix}0\\1\\0\end{pmatrix}=\begin{pmatrix}0\\1\\0\end{pmatrix}$ and $T\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$.

Now the standard transformation matrix is:

$$\begin{pmatrix}T\begin{pmatrix}1\\0\\0\end{pmatrix} & T\begin{pmatrix}0\\1\\0\end{pmatrix} & T\begin{pmatrix}0\\0\\1\end{pmatrix}\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&0\end{pmatrix}$$