

$\begin{pmatrix} 2 & 5 & -3 & g \\ 4 & 7 & -4 & h \\ -6 & -3 & 1 & k \end{pmatrix}$, then the Corresponding system of equations would be

$$\begin{aligned} 2x + 5y - 3z &= g \\ 4x + 7y - 4z &= h \\ z - 3y - 6x &= k \end{aligned}$$

But we will deal with the already given augmented matrix and will apply the elementary row operations on this as follows;

$$\text{By } R'_2 \longrightarrow R_2 - 2R_1, R'_3 \longrightarrow R_3 + 3R_1$$

$$\begin{pmatrix} 2 & 5 & -3 & g \\ 0 & -3 & 2 & h - 2g \\ 0 & 12 & -8 & 3g + k \end{pmatrix}$$

$$\text{By } R'_3 \longrightarrow R_3 + 4R_2$$

$$\begin{pmatrix} 2 & 5 & -3 & g \\ 0 & -3 & 2 & h - 2g \\ 0 & 0 & 0 & 4h - 5g + k \end{pmatrix},$$

Now the corresponding system of equation will become;

$$\begin{aligned} 2x + 5y - 3z &= g \\ 2z - 3y &= h - 2g \\ 0 &= 4h - 5g + k \end{aligned}$$

So the last equation \implies as the LHS is zero, so the consistency of the system demands that RHS should also be zero.

$\therefore 4h - 5g + k = 0$ is the required equation(condition) under which the system will have a solution.