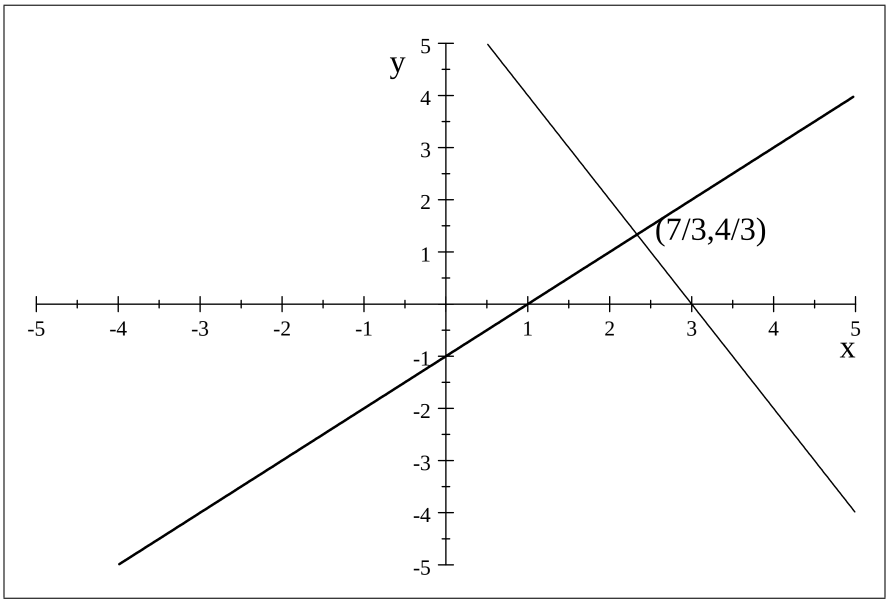
Question:
solve the linear system

$$\begin{array}{c} x - y = 1 \\ 2x + y = 6 \end{array}$$, Solution is:

Solution:

Method 1:(Graphical Method)

From the given system, if draw their graphs then we get the two straight line, whose point of intersection is $(x, y) = (\frac{7}{3}, \frac{4}{3})$, which is the required solution set of the system.



Method 2:(Crammer's Rule)

For the given system: $\begin{array}{c} x - y = 1 \\ 2x + y = 6 \end{array}$

$\Longrightarrow$its matrix form is:$AX = b \Longrightarrow \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$,

where $A = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$ is its associated coefficient matrix,

$X = \begin{pmatrix} x \\ y \end{pmatrix}$,

and its determinant= $\det(A) = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 1(1) - (-1)2 = 1 + 2 = 3.$

Now the required solution is given by the Crammer's rule as follows:

$$x = \frac{\det(\ b \quad Col_2 A\ )}{\det A} = \frac{\begin{vmatrix} 1 & -1 \\ 6 & 1 \end{vmatrix}}{3} = \frac{1(1)-(-1)6}{3} = \frac{1+6}{3} = \frac{7}{3}$$

$$y = \frac{\det(\ Col_1 A \quad b\ )}{\det A} = \frac{\begin{vmatrix} 1 & 1 \\ 2 & 6 \end{vmatrix}}{3} = \frac{1(6)-(1)2}{3} = \frac{6-2}{3} = \frac{4}{3}$$

$\therefore X = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{7}{3} \\ \frac{4}{3} \end{pmatrix}$

Method 3:(Inverse Matrix Method)

For the given system: $\begin{matrix} x - y = 1 \\ 2x + y = 6 \end{matrix}$,

its matrix form is:$AX = b \Longrightarrow \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$,

where $A = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$ is its associated coefficient matrix,

$X = \begin{pmatrix} x \\ y \end{pmatrix}$,

and its determinant= $\det(A) = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 1(1) - (-1)2 = 1 + 2 = 3.$

Now by Inverse method, if $AX = b \Longrightarrow X = A^{-1}b$,

where $A^{-1} = \frac{1}{\det A}\text{Adj}.A = \frac{1}{3}\begin{pmatrix} 1 & 1 \\ -2 & 1 \end{pmatrix}$.

$\therefore X = A^{-1}b$

$\Longrightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 6 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1(1) + 1(6) \\ -2(1) + 1(6) \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 7 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{7}{3} \\ \frac{4}{3} \end{pmatrix}$.

Method 4:(Echelon Form)

For the given system: $\begin{matrix} x - y = 1 \\ 2x + y = 6 \end{matrix}$,

its corresponding augmented matrix is:$\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & 6 \end{pmatrix}$.

Now we reduce it into Echelon form as follows;

$\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & 6 \end{pmatrix}$

By $R_2' \to R_2 - 2R_1$

$\sim \begin{pmatrix} 1 & -1 & 1 \\ 2 - 2(1) & 1 - 2(-1) & 6 - 2(1) \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 3 & 4 \end{pmatrix}$

Now the last row implies that $0x + 3y = 4 \Longrightarrow y = \frac{4}{3}$

and 1st row implies that $x - y = 1 \Longrightarrow x - \frac{4}{3} = 1 \Longrightarrow x = \frac{4}{3} + 1 = \frac{7}{3}$

$\therefore$required solution is $(x, y) = (\frac{7}{3}, \frac{4}{3})$

Method 5:(Reduce Echelon Form)

For the given system: $\begin{matrix} x - y = 1 \\ 2x + y = 6 \end{matrix}$,

its corresponding augmented matrix is:$\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & 6 \end{pmatrix}$.

Now we reduce it into Echelon form as follows;

$\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & 6 \end{pmatrix}$

By $R_2^{'} \rightarrow R_2 - 2R_1$

$$\sim \begin{pmatrix} 1 & -1 & 1 \\ 2-2(1) & 1-2(-1) & 6-2(1) \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 3 & 4 \end{pmatrix}$$

By $R_2^{'} \rightarrow \left(\frac{1}{3}\right) R_2$

$$\sim \begin{pmatrix} 1 & -1 & 1 \\ \frac{0}{3} & \frac{3}{3} & \frac{4}{3} \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & \frac{4}{3} \end{pmatrix}$$

By $R_1^{'} \rightarrow R_1 + R_2$

$\sim \begin{pmatrix} 1+0 & -1+1 & 1+\frac{4}{3} \\ 0 & 1 & \frac{4}{3} \end{pmatrix} = \begin{pmatrix} 1 & 0 & \frac{7}{3} \\ 0 & 1 & \frac{4}{3} \end{pmatrix}$ i.e. is the reduced Echelon form,

where the 1st row $\Longrightarrow 1x + 0y = \frac{7}{3}$ or $x = \frac{7}{3}$

and the 2nd row $\Longrightarrow 0x + 1y = \frac{4}{3} \Longrightarrow y = \frac{4}{3}$.

$\therefore$ required solution is $(x, y) = (\frac{7}{3}, \frac{4}{3})$