Practice Questions of Lecture 7 to 9

Practice Qs of Lecture 7:

Q #1: Solve the equation: $e^{2ix} = 0 + i0$, where $x \in \mathbb{R}$.

Q #2: Show that the period of e^{2ix} is $2\pi i$, where $x \in \mathbb{R}$.

Q #3:

If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2 \in \mathbb{C}$, then show that $e^{z_1} e^{z_2} = e^{x_1 + x_2}\left[\cos(y_1 + y_2) + i\sin(y_1 + y_2)\right]$.

Q #4: Show that $\left|e^{iz}\right| = e^{-y}$, for $z = (x + iy) \in \mathbb{C}$.

Practice Qs of Lecture 8:

Q #5: If $e^{2ix} = \cos 2x + i\sin 2x$, then show that $\sin 2x = \dfrac{e^{2ix} - e^{-2ix}}{2i}$.

Q #6: If $e^{2ix} = \cos 2x + i\sin 2x$, then show that $\cos 2x = \dfrac{e^{2ix} + e^{-2ix}}{2}$.

Q #7: Show that $\left\{\dfrac{n\pi}{2}\right\}_{n\in\mathbb{Z}}$ is the solution set of equation $\cos z = 0$.

Q #8: Prove that $1 + \tan^2 z = \sec^2 z$ for all $z \in \mathbb{C}$.

Practice Qs of Lecture 9:

Q #9: Prove that $\sin iy = i\sinh y$.

(Hint: Use $\sin y = \dfrac{e^{iy} - e^{-iy}}{2i}$)

Q #10: Prove that $\cosh iy = \cos y$.

(Hint. Use $\cosh y = \dfrac{e^{y} + e^{-y}}{2}$)

Q #11: Prove that $\cosh^2 x - \sinh^2 x = 1$.

Q #12: Show that $\coth x = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$.

Q #13: Show that $\operatorname{csch} x = \dfrac{2}{e^x - e^{-x}}$.