Practice Qs of Lecture 4 to 6

Q1. Find the all possible values of $\left(\cos\alpha + i\sin\alpha\right)^{\frac{1}{5}}$.

Q2. Show that $e^{i\frac{\pi}{12}}e^{i\frac{\pi}{4}}e^{i\frac{2\pi}{3}} = -1$.

Q3. Show that $\lim_{n\to\infty}\left(z_1.z_2.\dots z_n\right) = -1$, where $z_k = e^{i\frac{\pi}{2^k}}$, $k \in \mathbb{Z}$.

Q4. Prove that $\left(x+iy\right)^{\frac{s}{t}} + \left(x-iy\right)^{\frac{s}{t}} = 2\left(x^2+y^2\right)^{\frac{s}{2t}}\cos\left(\frac{s}{t}\tan^{-1}\frac{y}{x}\right)$

Hint (Use polar and Euler's forms).

Q5. By De Moivre's theorem, $\left(\cos x + i\sin x\right)^2 = \cos 2x + i\sin 2x$, then show that

$\cos 2x = \cos^2 x - \sin^2 x$

Q6. If $x - \frac{1}{x} = 2i\cos y$, then show that $x = \pm\sin y + i\cos y$.

Q7. Evaluate $(1+i)^{200}$.

Hint (Use polar form and De Moivre's Theorem).

Q8. Show that the sum of roots of equation: $x^4 - 4 = 0$ vanishes.

Q9. Find the four roots of i.

Answer Key:

Q1. $\left(\cos\alpha + i\sin\alpha\right)^{\frac{1}{5}} = \cos\left(\frac{\alpha+2k\pi}{5}\right) + i\sin\left(\frac{\alpha+2k\pi}{5}\right)$, $k \in \mathbb{Z}$

Q7. -2^{100}

Q9.

$$x_0 = \text{cis}\left(\left(4(0)+1\right)\frac{\pi}{8}\right) = cis\left(\frac{\pi}{8}\right)$$

$$x_1 = \text{cis}\left(\left(4(1)+1\right)\frac{\pi}{8}\right) = cis\left(\frac{5\pi}{8}\right)$$

$$x_2 = \text{cis}\left(\left(4(2)+1\right)\frac{\pi}{8}\right) = cis\left(\frac{9\pi}{8}\right)$$

$$x_3 = \text{cis}\left(\left(4(3)+1\right)\frac{\pi}{8}\right) = cis\left(\frac{13\pi}{8}\right)$$