Practice Questions of Lecture 38 to 43_Solution

Q.1: Identify the surface $x^2-4y^2-4x+8y-4z=0$ and $y-$ and $z-$intercepts.

Solution:

$$x^2-4y^2-4x+8y-4z=0$$
$$x^2-4y^2-4x+8y=4z$$
$$x^2-4x+4-4\left(y^2-2y+1\right)=4z$$
$$\frac{(x-2)^2}{4}-\frac{(y-1)^2}{1}=z$$

$\Rightarrow$ Hyperbolic paraboloid

$y-$**intercept:**

$$x=0,\ z=0\ \Rightarrow -\frac{(y-1)^2}{1}=0$$
$$y=1$$

$z-$**intercept:**

$x=0,\ y=0\ \Rightarrow z=0$

Q.2: Identify the surface $4x^2+25y^2-z^2-8x-50y+6z+138=0$ and trace in all coordinate planes.

Solution:

$$4\left(x^2-2x+1\right)+25\left(y^2-2y+1\right)-\left(z^2-6z+9\right)=-100$$
$$\frac{(x-1)^2}{25}+\frac{(y-1)^2}{4}-\frac{(z-3)^2}{100}=-1$$

Hyperboloid of two sheets

Trace in $yz-$**plane:** Put $x=0$

$$\Rightarrow \frac{1}{25}+\frac{(y-1)^2}{1}-\frac{(z-3)^2}{100}=-1$$
$$\frac{(y-1)^2}{1}-\frac{(z-3)^2}{100}=\frac{-24}{25}$$
$$\frac{(z-3)^2}{100}-\frac{(y-1)^2}{1}=\frac{24}{25}$$
$$\frac{(z-3)^2}{96}-\frac{(y-1)^2}{24/25}=1 \quad \Rightarrow \text{ Hyperbola}$$

Trace in $xy-$**plane:** Put $z=0$

$$\Rightarrow \frac{(x-1)^2}{25}+\frac{(y-1)^2}{4}-\frac{9}{100}=-1$$

$$\frac{(x-1)^2}{25}+\frac{(y-1)^2}{4}=-\frac{91}{100} \quad \Rightarrow \text{ No Trace}$$

Trace in $xz-$plane: Put $y=0$

$$\Rightarrow \frac{(x-1)^2}{25}+\frac{1}{4}-\frac{(z-3)^2}{100}=-1$$

$$\frac{(z-3)^2}{100}-\frac{(x-1)^2}{25}=\frac{5}{4} \quad \Rightarrow \text{ Hyperbola}$$

Q.3: In an equilateral triangle, show that $\sec B = 1+\sec b$.

Solution:

We know that

$$\cos b = \cos c \cos a + \sin c \sin a \cos B$$

$$\Rightarrow \cos B = \frac{\cos b - \cos c \cos a}{\sin c \sin a}$$

Since triangle is equilateral, $a=b=a$, hence

$$\cos B = \frac{\cos b - \cos^2 b}{\sin^2 b}$$

$$=\frac{\cos b(1-\cos b)}{1-\cos^2 b}=\frac{\cos b(1-\cos b)}{(1-\cos b)(1+\cos b)}=\frac{\cos b}{1+\cos b}$$

$$=\frac{1/\sec b}{1+1/\sec b}=\frac{1/\cancel{\sec b}}{\frac{\sec b+1}{\cancel{\sec b}}}=\frac{1}{1+\sec b}$$

Taking reciprocals on both sides,

$$\frac{1}{\cos B}=1+\sec b$$

$$\Rightarrow \sec B = 1+\sec b$$

Q.4: Prove that in a spherical triangle ABC

$$\sin\frac{A}{2}=\sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b \sin c}}, \ where \ 2s=a+b+c.$$

Solution:

As we know that

$$2\sin^2\frac{A}{2}=1-\cos A$$

$$=1-\frac{\cos a-\cos b\cos c}{\sin b\sin c}$$

$$=\frac{\sin b\sin c-\cos a+\cos b\cos c}{\sin b\sin c}$$

$$=\frac{\sin b\sin c+\cos b\cos c-\cos a}{\sin b\sin c}$$

$$=\frac{\cos(b-c)-\cos a}{\sin b\sin c} \qquad (\because \sin\alpha\sin\beta+\cos\alpha\cos\beta=\cos(\alpha-\beta))$$

$$=\frac{2\sin\frac{a+b-c}{2}\sin\frac{a-b+c}{2}}{\sin b\sin c} \qquad \left(\because \cos\alpha-\cos\beta=2\sin\frac{\beta+\alpha}{2}\sin\frac{\beta-\alpha}{2}\right)$$

Since $2s=a+b+c$

$$\Rightarrow s=\frac{a+b+c}{2}$$

$$\Rightarrow s-c=\frac{a+b+c}{2}-c \quad and \quad s-b=\frac{a+b-c}{2}-b$$

$$\Rightarrow s-c=\frac{a+b-c}{2} \qquad and \quad s-b=\frac{a-b+c}{2}$$

Hence $\quad 2\sin^2\frac{A}{2}=\frac{2\sin(s-c)\sin(s-b)}{\sin b\sin c}$

$$\Rightarrow \sin^2\frac{A}{2}=\frac{\sin(s-c)\sin(s-b)}{\sin b\sin c}$$

$$\Rightarrow \sin\frac{A}{2}=\sqrt{\frac{\sin(s-c)\sin(s-b)}{\sin b\sin c}}$$