Practice Questions of Lecture 29 to 34_Solution

Q.1: Find the nature of the origin on curve $x^3+y^3-3axy=0$.

Solution:

Here the lowest degree term in the equation is $3axy$. We will equate $3axy$ to zero to find the equation of the tangent at the origin. i.e $3axy=0 \Rightarrow x=0$ and $y=0$.

Hence the origin is either a node or an isolated point. When x is small, the equation of curve becomes

$$y^3-3axy=0 \text{ (neglecting } x^3\text{)},$$

$$\Rightarrow \quad y^3=3axy,$$

$$\Rightarrow \quad y^2=3ax,$$

which represents two real branches $y=\sqrt{3ax}$ and $y=-\sqrt{3ax}$. Hence the origin is a node.

Q.2: Find the trace of the surface $x^2-y^2+3z^2+2xy+xz=0$ with $xy-\text{plane}$, $yz-\text{plane}$ and $zx-\text{plane}$.

Solution:

For trace of the surface with $xy-\text{plane}$, put $z=0$. We get,

$$x^2-y^2+2xy=0.$$

For trace of the surface with $yz-\text{plane}$, put $x=0$. We get,

$$-y^2+3z^2=0.$$

For trace of the surface with $zx-\text{plane}$, put $y=0$. We get,

$$x^2+3z^2+xz=0.$$

Q.3: Check the symmetry of the surface $x^2+z^2-4xy+xz=0$.

Solution:

***x*-axis:**

$$f(x,-y,-z)=x^2+(-z)^2-4x(-y)+x(-z),$$
$$=x^2+z^2+4xy-xz \neq f(x,y,z),$$

$\therefore$ Surface is not symmetric about $x-\text{axis}$.

***y*-axis:**

$$f(-x,y,-z)=(-x)^2+(-z)^2-4(-x)(y)+(-x)(-z),$$
$$=x^2+z^2+4xy+xz \neq f(x,y,z),$$

$\therefore$ Surface is not symmetric about $y-\text{axis}$.

***z*-axis:**

$$f(-x,-y,z)=(-x)^2+z^2-4(-x)(-y)+(-x)(z),$$
$$=x^2+z^2-4xy-xz \neq f(x,y,z),$$

$\therefore$ Surface is not symmetric about $z-\text{axis}$.

Q.4: Find all intercepts of the surface $x^2+2y^2-3z^2+2xy+x-2y+4z=0$.

Solution:

For x-intercept, put y = 0 and z = 0, we get:

$$x^2+x=0,$$
$$\Rightarrow x(x+1)=0 \Rightarrow x=0,\ \ x=-1.$$

For y –intercept, put x = 0, z = 0, we get:

$$2y^2-2y=0,$$
$$\Rightarrow 2y(y-1)=0 \Rightarrow y=0,\ \ y=-1.$$

For z-intercept, put x = 0 and y = 0, we get:

$$-3z^2+4z=0,$$
$$\Rightarrow z(-3\mathrm{z}+4)=0,$$
$$\Rightarrow \mathrm{z}=0,\ \mathrm{z}=\frac{4}{3}.$$

Q.5: Find the equation of sphere with center (2, -1, 5) and diameter = 4.

Solution:

$$\text{Radius}=2.$$

So equation of sphere is:

$$(x-2)^2+(y+1)^2+(z-5)^2=(2)^2,$$
$$\Rightarrow x^2-4x+4+y^2+2y+1+z^2-10z+25=4,$$
$$\Rightarrow x^2+y^2+z^2-4x+2y-10z+26=0.$$

Q.6: Find the equation of sphere with center (2, 1, 3) and tangent to the plane $2x+y+z=2$.

Solution:

As the distance between center and plane is equal to the radius of sphere so

$$r=\left|\frac{2(2)+1(1)+1(3)-2}{2^2+1^2+1^2}\right|=\frac{6}{6}=1.$$

Hence equation of sphere is:

$$(x-2)^2+(y-1)^2+(z-3)^2=1^2,$$
$$\Rightarrow x^2+y^2+z^2-4x-2y-6z+13=0.$$