Practice Questions of Lecture 26 to 28_Solution

Q.1: Determine the interval where $f(\mathrm{x})=\frac{5}{2}x^4+\frac{20}{3}x^3-40x^2$ is increasing on [-6, 4].

Solution:

First we will find the critical points.

Put $f^{/}(\mathrm{x})=0,$

$\Rightarrow 10x^3+20x^2-80x=0,$

$\Rightarrow 10x(x^2+2x-8)=0,$

$\Rightarrow x=0,\ x^3+2x-8=0,$

$\Rightarrow x=0,\ x=2,\ x=-4.$

Hence critical points are $x=0,\ x=2,\ x=-4$. These critical points divide the interval $[-6,4]$ into four subintervals which is $(-6,-4),\ (-4,0),\ (0,2),\ (2,4).$

Put $x=-1$ from $(-4,0)$ in $f^{/}(x),$

$f^{/}(-1)=90>0.$

Hence $f(x)$ is increasing on $(-4,0).$

Put $x=-5$ from $(-6,-4),$

$f^{/}(-5)=-350<0.$

Hence $f(x)$ is decreasing on $(-6,-4).$

Put $x=3$ from $(2,4),$

$f^{/}(3)=210>0.$

Hence $f(\mathrm{x})$ is increasing on $(2,4).$

Put $x=5$ from $(4,6),$

$f^{/}(5)=1350>0.$

Hence $f(x)$ is increasing on $(4,6).$

Q.2: Apply the second derivative test to determine the local maximum and local minimum values of the function $f(\mathrm{x})=2\mathrm{x}^3-12x^2+18x$.

Solution:

Put $f^{/}(x)=0,$

$\Rightarrow 6x^2-24x+18=0,$

$\Rightarrow x^2-4x+3=0,$

$\Rightarrow x=1,\ x=3$ are critical points.

Here $f^{//}(x)=12x-24$

so $f^{//}(1)=12-24=-12<0\Rightarrow f(x)$ has a local maximum at $x=1.$

$f^{//}(3)=12>0\Rightarrow f(x)$ has a local minimum at $x=3.$

Q.3: Calculate the maximum and minimum values of $f(x) = -20x + 5x^2$. Investigate whether there exist any relative minima or maxima outside the given interval [-1, 4].

Solution:

Put $f^{/}(\text{x}) = 0,$

$\Rightarrow -20 + 10x = 0,$

$\Rightarrow 10(x-2) = 0,$

$\Rightarrow x = 2$ is critical point.

The maxima and minima lie on the critical point $x = 2$ or at the end points of interval $x = -1, x = 4.$

$f(-1) = 25,$

$f(2) = -20,$

$f(4) = 0,$

Hence maxima is 25 at $x = -1$ and minima is -20 at $x = 2.$

Q.4: Determine the interval on which $f(x) = 2x^3 + 3x^2 - 36x$ is increasing or decreasing.

Solution:

Put $f^{/}(\text{x}) = 0,$

$\Rightarrow 6x^2 + 6x - 36 = 0,$

$\Rightarrow 6(x^2 + x - 6) = 0,$

$\Rightarrow x = 2,\ x = -3$ are critical points.

Testing:

If $x < 2,$

$f^{/}(1) = 6(1+1-6) = -24 < 0$

so $f(x)$ is decreasing on $(-\infty, 2).$

If $2 < x < 3,$

$f^{/}(2.5) = 6(2.5^2 + 2.5 - 6) = 16.5 > 0,$

so $f(x)$ is increasing on $(2, 3).$

If $x > 3,$

$f^{/}(4) = 6(4^2 + 4 - 6) = 6(14) > 0,$

so $f^{/}(x)$ is increasing on $(3, +\infty).$