Practice Questions of Lecture 23 to 24_Solution

Q.1: Find the horizontal and vertical asymptotes of the graph of $f(x)=\dfrac{2x+1}{x^3-1}$.

Solution:

For vertical assymptotes, put

$$x^3-1=0$$
$$\Rightarrow x=1$$

Hence $x=1$ is vertical asymptote.

For horizontal asymptote we take limit as $\lim_{x\to\infty} f(x)$. Hence,

$$\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\frac{2x+1}{x^3-1}$$
$$=\lim_{x\to\infty}\frac{\frac{2x}{x^3}+\frac{1}{x^3}}{1-\frac{1}{x^3}} \qquad \text{(divide the numerator and denominator by } x^3\text{)}$$
$$=\frac{0+0}{1-0}=0$$

Hence $y=0$ is horizontal asymptote.

Q.2: Discuss the asymptote of following rational function:

$$g(x)=\frac{x^2+\frac{1}{2}x}{3x+2}.$$

Solution:

Here $g(x) = \dfrac{x^3 + \frac{1}{2}x}{3x+2}$

we will do division of rational function.

$$
\begin{array}{r|l}
 & \frac{1}{3}x - \frac{1}{18} \\
3x+2 & x^2 + \frac{1}{2}x \\
 & x^2 + \frac{2}{3}x \\
 & - \quad - \\
\hline
 & -\frac{1}{6}x \\
 & -\frac{1}{6}x - \frac{1}{9} \\
 & + \qquad + \\
\hline
 & 0
\end{array}
$$

So $g(\mathrm{x}) = \frac{1}{3}x - \frac{1}{18} + \dfrac{1/9}{3x+2}$

As $x \to \infty$, $\dfrac{1/9}{3x+2} \to 0$, therefore $\frac{1}{3}x - \frac{1}{18}$ is slant asymptote.

Also for vertical asymptote, put

$$3x + 2 = 0 \Rightarrow x = -\frac{2}{3}$$

Hence $x = -\frac{2}{3}$ is vertical asymptote.

Q.3: Find the slant asymptote of the function $f(x) = \dfrac{3x^3 + x^2 + 5}{2 - x^2}$.

Solution:

Here $f(x)=\dfrac{3x^3+x^2+5}{2-x^2}$

After long division, we get:

$f(x)=-3x-1+\dfrac{6x+7}{2-x^2}$

As $x\to\infty,\ \dfrac{6x+7}{2-x^2}\to 0, so -3x-1$ is slant asymptote.

Q.4: Find the asymptote of the curve $r\sin\theta=3\cos 2\theta$.

Solution:

$r\sin\theta=3\cos 2\theta$

$\dfrac{1}{r}=\dfrac{\sin\theta}{3\cos 2\theta}$

$u=\dfrac{\sin\theta}{3\cos 2\theta}$..............................(1)

put $u=0\Rightarrow\sin\theta=0\Rightarrow\theta=n\pi,\ n\in\mathbb{Z}$

Now, differentiate (1) w.r.t θ,

$\dfrac{du}{d\theta}=\dfrac{1}{3}\dfrac{\cos 2\theta\cos\theta+2\sin\theta\sin 2\theta}{\cos^2 2\theta}$

$\Rightarrow\lim\limits_{\theta\to n\pi}\dfrac{du}{d\theta}=\dfrac{1}{3}\dfrac{\cos 2n\pi\cos n\pi+2\sin n\pi\sin 2n\pi}{\cos^2 2n\pi}$

As $\cos 2n\pi=1,\ \sin n\pi=0, \sin 2n\pi=0$ *and* $\cos n\pi=(-1)^n$, *so*

$\lim\limits_{\theta\to n\pi}\dfrac{du}{d\theta}=\dfrac{(-1)^n}{3}$

We know that

$P=-\lim\limits_{\theta\to n\pi}\left(\dfrac{d\theta}{du}\right)$, *so*

$-\dfrac{3}{(-1)^n}=r\sin(n\pi-\theta)$

$=r[\sin(n\pi)\cos\theta-\cos(n\pi)\sin\theta]$

$=r[(-1)^n\sin\theta]$

$-3=(-1)^n r[(-1)^n\sin\theta]$

$3=r\sin\theta$

Q.5: Find the absolute maximum and absolute minimum values of $f(x)=x^3-3x^2-9x$ on the interval [-2, 4] and determine where these values occur.

Solution:

First we will critical points of given function.

For critical point put

$$f^{/}(\mathrm{x}) = 0$$
$$\Rightarrow 3x^2 - 6x - 9 = 0$$
$$\Rightarrow x^2 - 2x - 3 = 0$$
$$\Rightarrow x^2 - 3x + x - 3 = 0$$
$$\Rightarrow x(x-3) + (x-3) = 0$$
$$\Rightarrow x = -1,\ x = 3$$

Now we find the value of the function at critical points and at the end points of interval [-2,4].

$f(3) = -27$

$f(-1) = 5$

$f(-2) = -2$

$f(4) = -20$

Hence absolute maximum $= 5$ and absolute minimum $= -20$