Practice Questions of Lecture 13 to 15_Solution

Q #1: Convert the following equation into the standard form of circle and then identify the center and radius: $x^2+y^2+4x-4y+4=0$

Answer.: $\text{center}:(2,-2);\ \text{radius}=2$

Solution:

$$\because \quad x^2+y^2+4x-4y+4=0$$
$$\Rightarrow \left(x^2+4x\right)+\left(y^2-4y\right)=-4,$$
By completing the square we have
$$\Rightarrow \left(x^2+4x+4\right)+\left(y^2-4y+4\right)=-4+4+4,$$
$$\Rightarrow (x+2)^2+(y-2)^2=4,$$
$$\Rightarrow (x+2)^2+(y-2)^2=(2)^2,$$
$$\therefore\ \text{center}:(-2,2),\ \text{radius}=2.$$

Q #2: Convert the following equation into the standard form of circle and then identify the center and radius: $x^2+y^2-3y-4=0$.

(Answer.: $\text{center}:\left(0,\frac{3}{2}\right),\ \text{radius}=\frac{5}{2}$)

Solution:

$$\because \quad x^2+y^2-3y-4=0,$$
$$\Rightarrow x^2+\left(y^2-3y\right)=4,$$
By completing the square we have
$$\Rightarrow x^2+\left(y^2-3y+\frac{9}{4}\right)=4+\frac{9}{4},$$
$$\Rightarrow (x-0)^2+\left(y-\frac{3}{2}\right)^2=\frac{25}{4},$$
$$\Rightarrow (x-0)^2+\left(y-\frac{3}{2}\right)^2=\left(\frac{5}{2}\right)^2,$$
$$\therefore\ \text{center}:\left(0,\frac{3}{2}\right),\ \text{radius}=\frac{5}{2}.$$

Q #3: Write the equation of circle if center is at (-1, 2) and diameter is 8.

(Ans. $x^2+y^2+2x-4y-11=0$)

Solution:

$\because$ The equation of a circle with center at (h,k) and a radius of r is

$(x-h)^2+(y-k)^2=r^2,$

$\because$ Here $h=-1,\ k=2,\ r=4,\ \left(\because d=8\Rightarrow r=\dfrac{d}{2}=\dfrac{8}{2}=4\right),$

$\Rightarrow (x-h)^2+(y-k)^2=r^2,$

$\Rightarrow (x-(-1))^2+(y-2)^2=(4)^2,$

$\Rightarrow (x+1)^2+(y-2)^2=16,$

$\Rightarrow x^2+2x+1+y^2-4y+4-16=0,$

$\Rightarrow x^2+y^2+2x-4y-11=0.$

Q #4: Find the vertex and focus of the given parabola $(x-2)^2=5(y+2)$.

Solution:

Here we have $(x-2)^2=5(y-(-2))$. Compare it with standard form of parabola which is

$(x-h)^2=4a(y-k)^2,$

$\Rightarrow h=2,\ k=-2$ and $4a\Rightarrow a=\dfrac{5}{4},$

Hence vertex: $(h,k)=(2,-2).$

Since $a=\dfrac{5}{4}>0$ so parabola open upwards. Therefore coordinate of focus will be:

Focus: $(h,k+a)=\left(2,-2+\dfrac{5}{4}\right)=\left(2,-\dfrac{3}{4}\right).$

Q #5: Find the vertex and focus of the given parabola $(y-2)^2=-8(x-3)$.

Solution:

Here we have $(y-2)^2 = -8(x-3)$. Compare it with standard form of parabola which is

$\left(y-k\right)^2 = 4a(x-h)^2,$

$\Rightarrow h = 3,\ k = 2$ and Lectusrectum $= 4a = -8 \Rightarrow a = -2.$

Hence vertex: $(h,k) = (3,2).$

Since $a = -2 < 0$, so parabola open leftwards. Therefore coordinate of focus will be:

Focus: $(h+a,k) = \left(3-2,2\right) = \left(1,2\right).$

Q #6: Represent the following equation in standard equation of parabola and then find the vertex and focus:

$$x^2 + 8x - 4y + 8 = 0\ .$$

Solution:

$x^2 + 8x - 4y + 8 = 0.$

Now completing the square, we get

$x^2 + 2(x)(4) + (4)^2 = 4y + 8 + (4)^2,$

$\left(x+4\right)^2 = 4y + 24,$

$\left(x+4\right)^2 = 4(y+6).$ ____ (1)

Compare eq.(1) with standard form $(x-h)^2 = 4a(y-k)$, we have :

$h = -4, k = -6,$

and latus rectum $= 4\text{a} = 4,$

$$\Rightarrow \text{a} = 1,$$

Vertex : $(h,k) = (-4,-6).$

Since $a = 1 > 0$, so parabola opens upward. Therefore coordinate of focus will be:

Focus : $(h, k+a) = (-4, -6+1) = (-4,-5).$

Q #7: Find the center and foci of the following ellipse.

(a) $\dfrac{\left(x-1\right)^2}{3^2} + \dfrac{\left(y-3\right)^2}{2^2} = 1$

(b) $\dfrac{\left(x+4\right)^2}{25} + \dfrac{\left(y-1\right)^2}{16} = 1$

(c) $\dfrac{\left(x-5\right)^2}{49} + \dfrac{\left(y+2\right)^2}{64} = 1$

(a) $\dfrac{(x-1)^2}{3^2}+\dfrac{(y-3)^2}{2^2}=1$

Solution:

Here $\dfrac{(x-1)^2}{3^2}+\dfrac{(y-3)^2}{2^2}=1$, coordinate of center is $(1,3)$.

Now $f=\sqrt{a^2-b^2}$. Here $a=3$ and $b=2$, so $f=\sqrt{9-4}=\sqrt{5}$.

Since major axis lies on x-axis, therefore, foci are:

$(1+\sqrt{5},3)$ and $(1-\sqrt{5},3)$.

(b) $\dfrac{(x+4)^2}{25}+\dfrac{(y-1)^2}{16}=1$

Solution:

Here $\dfrac{(x+4)^2}{25}+\dfrac{(y-1)^2}{16}=1$, coordinate of center is $(-4,1)$.

Now $f=\sqrt{a^2-b^2}$. Here $a^2=25$ and $b^2=16$, so $f=\sqrt{25-16}=\sqrt{9}=3$.

Since major axis lies on x-axis, therefore foci are:

$(-4+3,1)=(-1,1)$ and $(-4-3,1)=(-7,1)$.

(c) $\dfrac{(x-5)^2}{49}+\dfrac{(y+2)^2}{64}=1$

Solution:

Here $\dfrac{(x-5)^2}{49}+\dfrac{(y+2)^2}{64}=1$, coordinate of center is $(5,-2)$.

Now $f=\sqrt{a^2-b^2}$. Here $a^2=64$ and $b^2=49$, so $f=\sqrt{64-49}=\sqrt{15}$.

Since major axis lies on y-axis, therefore foci are:

$(5,-2+\sqrt{15})$ and $(5,-2-\sqrt{15})$.