Practice Exercise For Lecture 1

Q1. Show that $i^{2n-1}+i^{2n}+i^{2n+1}+i^{2n+2}=1-i$, where $n\in E$ =set of even positive integers.

Q2. Evaluate $\frac{3}{i}-\frac{i}{3}$.

Q3. Discuss how many anti-clock quarter rotations, the following complex numbers will have?
$-5i,\ -6,\ 8i$

Q4. Simplify $i^{11}+i^{40}+i^{30}$.

Q5. Discuss why $i\neq 0$.

Q6. Find the principle and all other possible arguments of $1+i$.

Q7. Express $-1-\sqrt{3}i$ into polar form.

Q8. Solve for $x=\sqrt{-3\sqrt{-3\sqrt{-3...\infty}}}$ and show that difference of its roots is pure imaginary.

Q9. For a complex number $z\in C$ if, $|z|=3$ and $Arg(z)=\frac{\pi}{3}$, then find $\frac{1}{\bar{z}}$.

Q10. Let $|z_1|=2$ and $z_2=-1+\sqrt{3}i$. Then by using Triangular inequality, find the extreme values of $\left|\frac{z_1+z_2}{2}\right|$.

Q11. Show that the locus of point $P(z)$ satisfying, $\left|\frac{1+i\bar{z}}{\bar{z}+1}\right|=2$ is

$3x^2+3y^2+8x-2y+3=0$.

<u>Answer Key</u>

Q2. $-\frac{10i}{3}$

Q3. (i) one clockwise quarter rotation
(ii) two anticlockwise rotaion
(iii) one anti-clockwise quarter rotation

Q4. $-i$

Q6. $Arg(z)=\frac{\pi}{4}$ and $\arg(z)=\frac{\pi}{4}+2n\pi,\ n\in Z$

Q7. $2\,cis(-\frac{2\pi}{3})$

Q8. $x_1=\frac{1+\sqrt{11}i}{2}$ and $x_2=\frac{1-\sqrt{11}i}{2}$

Q9. $\frac{1}{18}(1+\sqrt{3}i)$

Q10. $\max\left\{\left|\frac{z_1+z_2}{2}\right|\right\}=\frac{3}{2}$ and $\min\left\{\left|\frac{z_1+z_2}{2}\right|\right\}=\frac{1}{2}$