GDB No.1 Spring 2017 MTH403_Soluiton (Lecture # 1 to 10)

Question:

Prove that $\tanh^{-1} y = \sinh^{-1}\left(\dfrac{y}{\sqrt{1-y^2}}\right)$.

Solution:

Let $\tanh^{-1} y = u$,

then $y = \tanh u$,

Now
$$\frac{y}{\sqrt{1-y^2}} = \frac{\tanh u}{\sqrt{1-\tanh^2 u}} ,$$
$$= \frac{\tanh u}{\sqrt{\operatorname{sech}^2 u}} , \quad (\because\ 1-\tanh u = \operatorname{sech} u)$$
$$= \frac{\tanh u}{\operatorname{sech} u} ,$$
$$= \frac{\dfrac{\sinh u}{\cosh u}}{\dfrac{1}{\cosh u}} , \quad \left(\because\ \tanh u = \frac{\sinh u}{\cosh u} \text{ and } \operatorname{sech} u = \frac{1}{\cosh u}\right)$$
$$= \frac{\sinh u}{\cosh u} \times \frac{\cosh u}{1} ,$$
$$\Rightarrow \frac{y}{\sqrt{1-y^2}} = \sinh u ,$$
$$\Rightarrow \sinh^{-1}\left(\frac{y}{\sqrt{1-y^2}}\right) = u ,$$
$$\therefore \sinh^{-1}\left(\frac{y}{\sqrt{1-y^2}}\right) = \tanh^{-1} y, \quad (\because\ \tanh^{-1} y = u)$$

Hence $\tanh^{-1} y = \sinh^{-1}\left(\dfrac{y}{\sqrt{1-y^2}}\right)$ Proved.