Section 6.3
Annihilator Method

June 25, 2009

A Summary of This Session:

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(1) General Rules for finding particular solution involving sin and cos.

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Method of Undertermined Coefficients (rule p. 200)

To find a particular solution for

$$ay'' + by' + cy = P_m(t)e^{r\,t}$$

where $P_m(t)$ is a polynomial of degree m, use the form

$$y_p = t^s\ (A_0 + A_1\,t + \ldots + A_m\,t^m)\ e^{r\,t}$$

where $s = 0$ if r is not a root of the characteristic equation; $s = 1$ if r is a simple root of the characteristic equation; and $s = 2$ if r is a double a root of the characteristic equation.

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Rule p. 200 cont'd

To find a particular solution for

$$ay'' + by' + cy = P_m(t)\, e^{\alpha\, t} \cos \beta\, t + Q_m(t) e^{\alpha\, t} \sin \beta\, t$$

where $P_m(t)$ and $Q_n(t)$ are polynomials of degrees m and n, use the form

$$y_p = t^s \,\left(A_0 + A_1\, t + \ldots + A_k\, t^k\right)\, e^{\alpha\, t} \cos \beta\, t$$

$$+ t^s \,\left(B_0 + B_1\, t + \ldots + B_k\, t^k\right) e^{\alpha\, t} \sin \beta\, t.$$

Here $k = \max(m, n)$, $s = 0$ if $\alpha + i\beta$ is not a root of the characteristic equation; and $s = 1$ if $\alpha + i\beta$ is a root of the characteristic equation.

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Write down the general form of a particular solution to the equation

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The roots of the characteristic equation are: $r = -1 \pm i$. Note that $r = 1 + i$ is what one gets out of the forcing function, so $s = 1$. The highest polynomial is of degree $k = 3$. We try

$$y_p = t\,\left(A_0 + A_1\,t + \ldots + A_3\,t^3\right)\,e^{-t}\cos t$$
$$+t\,\left(B_0 + B_1\,t + \ldots + B_3\,t^3\right)e^{-t}\sin t.$$

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$$y'' - 4y' + 4y = 0.$$

We adopt the notation $D = \frac{d}{dx}$. So $D^2 = \frac{d^2}{dx^2}$. We can write this equation symbolically as:

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The expression $A = D^2 - 4D + 4 = (D - 2)^2$ is called a *linear differential operator*.

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The annihilator of $f = \cos \beta t$ and $g = \sin \beta t$ is $A = (D^2 + \beta^2)$.

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Example 3: What is the differential equation satisfied by $f = e^{2t} \cos 3t$. Calculate:
$f = 2e^{2t} \cos 3t - 3e^{2t} \sin 3t = 2f - 3e^{2t} \sin 3t$.

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$$f' - 2f = -3e^{2t} \sin 3t$$

and

$$f'' - 2f' = -9e^{2t} \cos 3t - 6e^{2t} \sin 3t = -9f + 2(f' - 2f)$$

We conclude, after simplification, that

$$f'' - 4f' + 13f = 0$$

or

$$(D^2 - 4D + 13)f = 0.$$

Complete the square

$$\left((D-2)^2 + 9\right) \, f = 0$$

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How to justify this, without going through a long calculation, as above? We note that the solutions to the differential equation

$$(D-5)^3 y = 0$$

are given by (since 5 is a triple root)

$$y = c_1 e^{5t} + c_2 te^{5t} + c_3 t62e^{5t}.$$

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So: e^{5t}, $t\,e^{5t}$, and $t^2\,e^{5t}$ are all annihilated by $(D-5)^3$.

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Example 7: Show that $(D+1)^2(D^2-2D+5)$ is an annihilator for $te^{-t} + e^t \sin 2t$.

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Answer: $y = c_1 e^{-t} + c_2 te^{-t} + c_3 e^t \cos 2t + c_4 e^t \sin 2t$.

Using Annihilator Method to Solve a DFQ

We want to solve $Ay = f$. We find an annihilator B for f, so $B\,f = 0$, and solve $BAy = 0$. **Important Note:** This will give the correct form of the particular solution all the time.

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(b) Use the annihilator method to find the form of the particular solution for

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(b) We note that $A = (D^2 - 1) = (D-1)(D+1)$.

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That is

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$$D(D^2 + 9)(D - 3)(D - 5)\, y = 0.$$

So the general solution is given by:

$$y = c_1 + c_2 \cos 3t + c_3 \sin 3t + c_4 e^{3t} + c_5 e^{5t}.$$

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Sine e^{3t} and e^{5t} are solutions of the homogeneous equation, the general form of the particular solution is:

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We need to factor the operator. Graph the function $y = x^3 - 2x^2 - 5x + 6$. What do you observe? Indeed $x = 1$ is one of the roots. We divide to obtain:

$$(D^3 - 2D^2 - 5D + 6) = (D-1)(D^2 - D - 6) = (D-1)(D+2)(D-3)$$

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The solution is of the form:

$$y = c_1 + c_2 x + c_3 e^{-3x}\cos 2x + c_4 e^{-3x}\sin 2x +$$

$$c_5 x\, e^{-3x}\cos 2x + c_6 x\, e^{-3x}\sin 2x + c_7 e^{2x}.$$