After determining the singular points, divide the coefficients of $\frac{d^2y}{dx^2}$ of the given DE i.e. by the coefficients of $\frac{dy}{dx}$ and y to form $P(x)$ and $Q(x)$ respectively.

So in the given example,

$\left(x^2-1\right)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-9y=\sin x$

So $P(x)=\frac{x}{(x^2-1)}=\frac{x}{(x-1)(x+1)}$ and $x=\pm 1$ are singular points and powers of $(x-1)\,(x+1)$ are at most one(maximum), so these $x=\pm 1$ are regular singular points of the given DE.

But for $\left(x^2-1\right)^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-9y=\sin x$,

$P(x)=\frac{x}{(x^2-1)^2}=\frac{x}{(x-1)^2(x+1)^2}$ and $x=\pm 1$ are singular points and powers of $(x-1)\,(x+1)$ are more than one, so these are are the irregular singular points of the given DE.