(MCQ, Lecture 16, Marks-01)

1)

If the roots of an auxiliary equation associated with the homogeneous differential equation are $\pm 2, 1 \pm 2i$,then its general solution is ——.

$y = Ae^{2x} + Be^{-2x} + e^{2ix}(C\cos x + D\sin x)$

$y = Ae^{2x} + Be^{-2x} + e^{ix}(C\cos 2x + D\sin 2x)$

$y = Ae^{2x} + Be^{-2x} + e^{x}(C\cos 2x + D\sin 2x)$ $(correct)$

$y = Ae^{2x} + Be^{-2x} + e^{2ix}(C\cos x - D\sin x)$

(MCQ, Lecture 17, Marks-01)

2)

If $a\cos x + b\sin x$ is the solution for $y'' + y = 0$, then which of the following would be general form of the particular solution for $y'' + y = 4\cos x - \sin x$?

$y_p = c\cos x + d\cos x$

$y_p = cx\cos x + dx\cos x \quad (correct)$

$y_p = e^x\left(c\cos x + d\cos x\right)$

$y_p = e^x\left(cx\cos x + dx\cos x\right)$

(MCQ, Lecture 18, Marks-01)

3)

If the annihilator of $e^{\pi x}$ and $\left(x^2 + 2x + 3\right)$ are $(D - \pi)$ and D^3 respectively, then the annihilator of their linear combination is —-

$(D - \pi) + D^3$

$(D - \pi) - D^3$

$D^3\left(D - \pi\right)(correct)$

$\frac{(D-\pi)}{D^3}$

(Descriptive, Lecture 16, Marks-02)

4)

If $y = e^{mx}$ is a solution of the differential equation:$\frac{d^2y}{dx^2} + ay = 0\,, a \in \mathbb{R}\,$, then construct its associated auxiliary equation.

Solution:

$\because y = e^{mx} \Longrightarrow \frac{dy}{dx} = \frac{d}{dx}e^{mx} = me^{mx} \quad \Longrightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(me^{mx}\right) = m^2e^{mx}$

$\therefore \frac{d^2y}{dx^2} + ay = 0 \Longrightarrow m^2e^{mx} + ae^{mx} = 0$

$e^{mx}\left(a + m^2\right) = 0 \Longrightarrow \boxed{m^2 + a = 0} \quad \because e^{mx} \neq 0$

i.e. the required auxiliary equation.

(Descriptive, Lecture 17, Marks-02)

5)

If $y = e^{-\frac{x}{2}}\left(a\cos\left(\frac{\sqrt{3}}{2}x\right) + b\sin\left(\frac{\sqrt{3}}{2}x\right)\right)$ is a complementary solution of the differential equation: $y'' + y = x\sin x$, then what will be the general form of its particular solution?

Solution:

Since $\sin\left(\frac{\sqrt{3}}{2}x\right)$ and $x\sin x$ are linearly independent, so the general form of its particular solution of the given D.E would be of the form:

$y_p = x(c\cos x + d\sin x)$.

(Descriptive, Lecture 18, Marks-02)

6)

If $(D-\pi)^2\,\pi x e^{\pi x} = 0$, $\left(D^2+\pi^2\right)\cos\pi x = 0$ and $D(\pi x) = 0$, then what will be the annihilator of their linear combination?

Solution:

Linear combination of the given functions: $c_1\pi x e^{\pi x} + c_2\cos\pi x + c_3\pi x$

$\Longrightarrow$its annihilator would be the product of annihilators of individual functions as:

$\left((D-\pi)^2\left(D^2+\pi^2\right)D\right)\left(c_1\pi x e^{\pi x} + c_2\cos\pi x + c_3\pi x\right) = 0$

$\therefore D\left(D^2+\pi^2\right)(D-\pi)^2$ is the required annihilator.

(Descriptive, Lecture 16, Marks-03)

7)

If $m^3+am=0$ is an associated auxiliary equation of the differential equation:$\frac{d^3y}{dx^3} + a\frac{dy}{dx} = 0, a\in\mathbb{R}$, then find its general solution.

Solution:

$\because$given that $m^3+am=0 \Longrightarrow m\left(a+m^2\right) = 0 \Longrightarrow m = 0 \vee m^2+a = 0$

$\Longrightarrow m = 0 \vee m^2+a=0 \Longrightarrow m = 0 \pm i\sqrt{a}$

$\therefore$the required general solution: $y = ae^{0x} + e^{0x}\left(b\cos\left(\sqrt{a}x\right) + c\sin\left(\sqrt{a}x\right)\right) = a + b\cos\left(\sqrt{a}x\right) + c\sin\left(\sqrt{a}x\right)$

(Descriptive, Lecture 17, Marks-03)

8)

What would be the general form of a particular solution of the differential equation:$y'' + y = 4\cos x - \sin x$?

Solution:

Associated homogenous D.E corresponding to given is: $y'' + y = 0$.

If $y = e^{mx}$ be its solution, then the auxiliary equation is:$m^2+1 = 0 \Longrightarrow m = 0 \pm i$

$\therefore$ the its general solution: $y_c = a\cos x + b\sin x$

$\because$ the input function contains $\sin x$ and $\cos x$ while complementary solution also contain this, which will no more be independent.

$\Longrightarrow$proposed general form of the particular solution is:

$\boxed{y_p = cx\cos x + dx\sin x}$

(Descriptive, Lecture 18, Marks-03)

9)

Determine the annihilator operator of general solution of the differential equation:$y'' + \pi y = 0$.

Solution:

Given that $y'' + \pi y = 0 \Longrightarrow \frac{d^2y}{dx^2} + \pi y = 0 \Longrightarrow \frac{d^2}{dx^2}y + \pi y = 0$

$\Longrightarrow D^2y + \pi y = 0 \qquad \because D^2 \equiv \frac{d^2}{dx^2}$
$\Longrightarrow \left(D^2 + \pi\right) y = 0$
$\Longrightarrow \left(D^2 + \pi\right)$ is the required annihilator the solution $y = f(x)$ of the given differential equation.

(Descriptive, Lecture 16, Marks-05)

10)

If $m^2 + 1 = 0$ is an auxiliary equation corresponding to the differential equation: $\frac{d^2y}{dx^2} + y = 0$, then find its particular solution satisfying the initial conditions: $y(0) = 1$ and $\frac{dy}{dx}|_{x=\pi} = -1$.

Note: $\frac{dy}{dx}|_{x=\pi} = -1$ means first derivative of y at $x = \pi$ is equal to -1.

Solution:

$\because m^2 + 1 = 0$ is given.$\Longrightarrow m = 0 \pm i$.
$\therefore$ the general solution is: $y = e^{0x}\left(a\cos x + b\sin x\right) = a\cos x + b\sin x$——(1)
$\Longrightarrow \frac{dy}{dx} = -a\sin x + b\cos x$——(2)
For given $y(0) = 1, (1) \Longrightarrow 1 = a\cos 0 + b\sin 0 \Longrightarrow a = 1$
and for $\frac{dy}{dx}|_{x=\pi} = -1, (2) \Longrightarrow -1 = -a\sin\left(\pi\right) + b\cos\left(\pi\right) \Longrightarrow b = 1$
$\therefore$ the required particular solution is: $\boxed{y_p = \cos x + \sin x}$

(Descriptive, Lecture 17, Marks-05)

11)

If the complementary solution of following non-homogenous differential equation is $ae^x + be^{2x}$, then determine its particular solution by using Undetermined Coefficients method.

$y'' - 3y' + 2y = 4e^x$.

Solution:

Here input function:$4e^x$ and particular solution also contains e^x.

$\therefore$ by linear independence, the proposed general form of the particular solution is: $y_p = cxe^x$

$\Longrightarrow y_p' = ce^x + cxe^x = ce^x\left(x + 1\right) \Longrightarrow y_p'' = ce^x\left(x + 2\right)$
Now the given: $y'' - 3y' + 2y = 4e^x$
$\Longrightarrow ce^x\left(x + 2\right) - 3ce^x\left(x + 1\right) + 2cxe^x = 4e^x$
$\Longrightarrow 2ce^x + cxe^x - 3ce^x - 3cxe^x + 2cxe^x = 4e^x$
$\Longrightarrow -ce^x = 4e^x \Longrightarrow \boxed{c = -4}$
$\therefore$the required particular solution:$\boxed{y_p = -4xe^x}$

(Descriptive, Lecture 18, Marks-05)

12)

If $L \equiv D^2 - 5D - 6$ is a linear differential operator such that $Ly = 0$,for $y = f(x)$. Then:

i) Construct a differential equation corresponding to L.
ii) Determine the general solution of differential equation in case of *(i)* .
iii) Determine the annihilator operator of the general solution in case of *(ii)* .

Solution:

i) Given that $Ly = 0 \Longrightarrow \left(D^2 - 5D - 6\right) y = 0 \Longrightarrow D^2y - 5Dy - 6y = 0$

$\because \frac{d}{dx} \equiv D$ is a differential linear operator$\Longrightarrow \frac{d^2}{dx^2} \equiv D$

$\therefore \frac{d^2}{dx^2}y - 5\frac{d}{dx}y - 6y = 0 \Longrightarrow \boxed{\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 6y = 0}$ is the required differential equation.

*ii)*Say $y = e^{mx}$ be its solution$\Longrightarrow \frac{dy}{dx} = me^{mx}$ and $\frac{d^2y}{dx^2} = m^2e^{mx}$

$\therefore$D.E$\Longrightarrow m^2e^{mx} - me^{mx} - 6e^{mx} = 0 \Longrightarrow e^{mx}\left(m+2\right)\left(m-3\right) = 0$

$\Longrightarrow \left(m+2\right)\left(m-3\right) = 0 \quad \because e^{mx} \neq 0$

$\Longrightarrow m = -2, 3$

$\therefore$the general solution: $y = ae^{-2x} + be^{3x}$

iii) $\because$Given that $Ly = 0 \Longrightarrow L \equiv D^2 - 5D - 6$ is the required annihilator operator for the general solution.