

A linear Differential Equation;

$a_n(x) \frac{d^n y}{dx^n} + a_{n-1}(x) \frac{d^{n-1} y}{dx^{n-1}} + \dots + \frac{dy}{dx} + a_0(x)y = F(x)$, where the coefficients $a_n(x), a_{n-1}(x), \dots, a_1(x), a_0(x)$ are the polynomials of x .

Now:

A point x_1 will be the ordinary point of the above linear DE, if at this point $a_n(x) \neq 0$ i.e. $a_n(x_1) \neq 0$ and if $a_n(x_1) = 0$, then x_1 is called the singular point of the above DE.

For example, for the 2nd order linear DE say $(x^2 - 1) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - 9y = \sin x$, here the coefficient of highest order derivative: $\frac{d^2 y}{dx^2}$ is $(x^2 - 1)$.

\implies ordinary points are $(x^2 - 1) \neq 0 \implies x^2 \neq 1 \implies x \neq \pm 1$. This means all the points except $x = \pm 1$ are the ordinary points of the DE and $x = \pm 1$ are the singular points.

Similarly for $\frac{d^2 y}{dx^2} + x \frac{dy}{dx} - 9y = \sin x$ or $x^0 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - 9y = \sin x$, there is no singular point as $x^0 \neq 0$ for any real value of x as $x^0 = 1$. Hence its ordinary points will be whole real line \mathbb{R} .

For $x \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - 9y = \sin x$, $x = 0$ is the only singular point and all other points are ordinary i.e. $\mathbb{R} - \{0\}$.